NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM

No. 1165

COEFFICIENT OF FRICTION, OIL FLOW AND HEAT BALANCE OF A FULL-JOURNAL BEARING

By P. I. Orloff

Aeronautical Engineering (Moscow) 9th year, Jan. 1935.

Washington
October 1947

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 1165

COEFFICIENT OF FRICTION, OIL FLOW AND HEAT BALANCE OF A FULL-JOURNAL BEARING*

By P. I. Orloff

1. COEFFICIENT OF FRICTION

Statement of the Problem. The friction between metal surfaces completely separated by a lubricant layer, or fluid friction, is the reaction of the layer to the motion of the surface. Experiment and theory show that fluid friction does not depend, or depends only to a slight extent, on the magnitude of the load on the surface and is determined to a very large extent by the viscosity of the lubricant and the velocity of the relative motion.

The usual method of expressing the friction force as a function of the load through the coefficient of friction is for fluid friction quite an artificial one and its adoption in the lubrication theory is merely a concession to the concepts of friction based on the laws of Coulomb.

The numerous experimental and theoretical investigations of the coefficient of friction of a journal bearing all express the coefficient as a function of the parameter $\eta\omega/k$, where η is the absolute viscosity of the oil, ω the angular velocity of the shaft and k the pressure. This magnitude which plays a large part in the discussion that follows we shall denote as the operating parameter of the bearing. For practical computations it is more convenient to make use of the related magnitude $\eta n/k$, where η is the speed of the shaft in revolutions per minute, k the pressure in kg/cm^2 on the projection of the bearing. For briefness $\eta n/k$ will be denoted by the symbol λ. The latter is connected with the magnitude $\eta\omega/k$ by the following relation

$$\lambda \frac{\text{centipoises rpm}}{\text{kg/cm}^2} = 9368 \cdot 10^5 \eta \frac{\text{kg sec/m}^2 \ \omega \ 1/\text{sec}}{k \ \text{kg/m}^2}$$

Petroff in 1883 proposed for the coefficient of friction of parallel surfaces separated by a lubricant layer of height h and moving relative to each other with velocity v the following expression:

---

*Aeronautical Engineering (Moscow) 9th year, Jan. 1935, pp. 25-56.

$$f = \frac{l}{h} \cdot \frac{\eta v}{k}$$

which follows immediately from the expression of Newton for the force required for the displacement of a viscous fluid between two parallel surfaces of solid bodies. According to the law of Newton the shear force T of a viscous fluid is proportional to the area, the viscosity of the fluid, and the velocity gradient across the fluid layer:

$$T = lb\ \eta\ \frac{dv}{dy} \tag{1}$$

where l is the length and b the width of the surface.

The shear force thus depends on the velocity profile across the oil layer. If the oil in the clearance is carried along only by the adhesion to the moving surface and the viscosity force the velocity gradient is constant and equal to v/h, the velocity profile is linear and equation (1) assumes the following form:

$$T = lb\ \eta\ \frac{v}{h} \tag{1a}$$

The coefficient of friction or the ratio of the frictional force T to the load on the bearing surfaces $P = klb$ in this case is equal to:

$$f = \frac{l}{h} \frac{\eta v}{k}$$

For a cylindrical shaft rotating concentrically with the angular velocity ω in a bearing with diametral clearance Δ the friction force by the law of Newton is equal to:

$$T = \pi\ dl\ \frac{\eta \omega r^2}{\Delta}$$

and the coefficient of friction

$$f = \frac{T}{kld} = \pi \frac{d}{\Delta} \frac{\eta\omega}{k} = \frac{\pi}{\psi} \frac{\eta\omega}{k} \tag{2}$$

where $\psi = \Delta/d$ is the relative clearance of the bearing. This is the formula of Petroff for a journal bearing. It was derived for a clearance of uniform width and takes into account only the friction of the viscous displacement of the oil and is therefore applicable only to the determination of the friction of lightly loaded and concentrically rotating shafts.

If a load is applied to the surfaces then, in addition to the reaction of the viscous displacement, the surfaces are subject to still another reaction of the oil flowing out under the pressure in the direction of the motion (fig. 2). For cylindrical bearings the zone of pressure constitutes generally 90 to 120°; in this zone forces are developed which increase the friction of the shaft above the value determined by the Petroff formula.

In determining the simultaneous reaction of the viscous displacement and potential flow of the oil in the loaded zone, assumed as 120°, Gümbel (reference 2) expressed the coefficient of friction as a function of the relative eccentricity of the shaft in the following manner:

$$f = 1.7 \sqrt{1-X} \tag{3}$$

where X is the relative eccentricity equal to the ratio of the absolute eccentricity of the shaft e to the radial clearance $\delta = \Delta/2$ (fig. 3). The relative eccentricity, as follows from the equation of discharge of Reynolds is a function of the magnitude $\eta\omega/k\psi^2$ that is

$$(\gamma)^* = \frac{\eta\omega}{k\psi^2} \tag{4}$$

Substituting in equation (3) for $1 - X$ the algebraic expression, represented in figure 5, of the relation between X and $\eta\omega/k\psi^2$, obtained by the numerical integration of a number of typical cases Gümbel obtained for the coefficient of friction the following expression:

$$f = 1.7\epsilon \sqrt{\frac{\eta\omega}{k}}$$

The coefficient ϵ that takes into account the inaccuracy of the algebraic expression for the relation between X and $\eta\omega/k\psi^2$ is shown in figure 1 as a function of $\eta\omega/k\psi^2$.

The eccentricity of the shaft in the bearing depends not only on the magnitude $\eta\omega/k\psi^2$ but also on the length of the bearing. The smaller the ratio l/d the more easily does the oil flow out of the clearance and the deeper must the shaft be set in the bearing in order to sustain the given load. To take into account the finite length of the bearing it was necessary for Gumbel to introduce in the above derived expression the coefficient of length of the bearing obtained from the working up of the tests of Lasche (reference 3). The final expression for the coefficient of friction according to Gumbel has the following form:

$$f = 1.7c'\sqrt{\frac{\eta\omega}{k}} \tag{5}$$

where $c' = \sqrt[4]{d/l + 1}$ is the correction for the finite length of the bearing. Assuming the value of ϵ equal on the average to 1 and choosing the ratio $l/d = 1$ Falz (reference 4) obtained the following widely known formula:

$$f = 3.8\sqrt{\frac{\eta\omega}{k}} \tag{6}$$

The above formula gives satisfactory results for partial bearings (without upper cover) with a bearing arc of about 120° but is unsuitable for full journal bearings particularly those with forced feed since the formula does not take into account the additional friction of the viscous displacement of the oil over the unloaded two thirds of the bearing. The formula may be used for the approximate estimate of the friction in full journal bearings only for small values of $\eta\omega/k$ where the additional friction of the viscous displacement is relatively small and the reaction of the flow from the loaded zone is the predominant factor. Moreover the range of values of $\eta\omega/k$ in which formula (6) gives satisfactory results is so near the critical value of $\eta\omega/k$ corresponding to the rupture of the oil film that it is practically impossible to use formula (6) for full journal bearings.

Test Data on the Coefficient of Friction. Test on the coefficient of friction generally determine the latter as an exponential function of η, ω, k. Illmer (reference 5) on carefully working up almost the entire literature on the coefficient of friction represents the latter in the following form:

$$f = K\frac{\eta^r v^m}{k^n}$$

where the exponents r, m, n, vary within the range 0.3 to 1 and K is a constant that takes into account the size of the clearance and other design factors. The rather complicated method proposed

628

by Illmer for determining the coefficient of friction is hardly applicable on account of its complexity and absence of any rational basis. We present the results of the experimental investigations of the various authors considered by Illmer (table 1) in Table 1.

Fomin (reference 6) in conducting tests with the crankshaft of an airplane engine found that r varies in the mean between the limits 0.3 to 0.4, m = 0.5 to 1, n = 0.6 to 1.

McKee (reference 7) investigating the coefficient of friction and its dependence on the clearance and length of bearing expressed the results of his tests by the following equation

$$f = \frac{\pi}{\psi} \cdot \frac{\eta\omega}{k} + \sigma a$$

the first term of which represents the formula of Petroff, a is a constant equal to 0.002, σ is a correction coefficient depending on the length of the bearing.

Coefficient of Friction of a Full Journal Bearing. The velocity distribution in the oil layer of a bearing working in the region of fluid lubrication with a clearance completely filled by the oil is schematically represented in figure 2 together with the diagram of the pressure over the circumference of the bearing. Over the entire circumference of the bearing the oil is carried along by the moving surface of the shaft and the velocity diagram across the oil layer had the form of a triangle the maximum ordinate of which is equal to the peripheral velocity of the shaft. The resistance of the oil, carried along by the motion of the shaft, per unit area is

$$\tau' = \eta \frac{dv}{dy} = \eta \frac{v}{h} \tag{7}$$

In the narrow part of the clearance due to the incompressibility of the oil a region of pressure is formed from which the oil flows out in the direction of motion and toward the ends of the bearing. The latter type of flow has no effect on the friction since the frictional force is directed at right angles to the peripheral velocity. The flow in the direction of motion however (along the X axis) gives additional friction. To the triangular velocity profile there is here added the parabolic profile of the potential flow, the equation for which, for a system of coordinates placed at the center of the oil layer, has the following form:

$$v = \frac{1}{2\eta}\left[y^2 - \left(\frac{h}{2}\right)^2\right]\frac{dp}{dx} \tag{8}$$

where dp/dx is the pressure gradient along the X axis.

The first derivative of the velocity with respect to y is equal to

$$\frac{dv}{dy} = \frac{y}{\eta}\frac{dp}{dx}$$

The friction force per unit area for the boundary surfaces, that is, for $y = \pm h/2$ is according to the law of Newton equal to

$$\tau'' = \eta\frac{dv}{dy} = \frac{h}{2}\frac{dp}{dx} \tag{8a}$$

The total friction of the bearing is made up of two components: of the viscous resistance of the oil T' carried along by the motion of the shaft over its entire periphery and of the reaction T'' of the flow of the oil from the loaded zone along the direction of the X axis:

$$T = T' + T'' = dl\ (\pi\tau + a\tau''),$$

where a is the loading angle of the bearing. The total coefficient of friction is the sum of the individual components

$$f = \frac{T''}{p} + \frac{T''}{p} = f' + f''$$

<u>T' and f'.</u> We shall determine the value of the first form of the friction according to equation (1a). We transform the linear coordinates of this equation into polar for which purpose we make use of figure 3 showing the shaft eccentrically placed in the bearing. We take the origin of coordinates on the line 0-0 connecting the centers of the shaft and bearing. For any angle φ the height of the clearance h can then be represented by the following expression:

$$h = r\psi\,(1 + X\cos\varphi), \tag{9}$$

where

r is the radius of the shaft

ψ the relative clearance, equal to Δ/d

X the relative eccentricity, equal to $2e/\Delta$

The total friction force over the entire periphery of the shaft is obtained by integrating expression (10) between the limits 2π to 0:

$$T'' = \frac{lr\eta\omega}{\psi}\int_0^{2\pi}\frac{d\varphi}{1 + X\cos\varphi} = \frac{lr\eta\omega}{\psi}\frac{2\pi}{\sqrt{1-X^2}}$$

The coefficient of friction f' is equal to

$$f' = \frac{T''}{kld} = \frac{\pi}{\psi}\frac{\eta\omega}{k}\frac{1}{\sqrt{1 - X^2}} = \frac{\pi}{\psi}\frac{\eta\omega}{k}(X)* \qquad (11)$$

We have thus obtained the formula of Petroff corrected by the factor $\frac{1}{\sqrt{1 - X^2}}$ that takes account of the eccentricity of the shaft in the bearing. For $X = 0$, that is, for a central setting of the shaft, formula (11) becomes the formula of Petroff.

Figure 4 for the function $(X)*$ shows that the correction term differs to an appreciable degree from 1 only for values $X > 0.5$ and that therefore the formula of Petroff is sufficiently accurate over the large range of practically occurring small eccentricities corresponding to high values of the parameter $\eta\omega/k$.

For convenience in using formula (11) we replace the eccentricity X by the magnitude $\eta\omega/k\psi^2$ of which it is a function. The relation between these magnitudes is given by the flow equation of Reynolds which in the interpretation of Gümbel (reference 9) for the cylindrical bearing for a mean value of the loading arc of 120° leads to the relation shown by the curves in figure 5. The lower curve refers to a bearing of infinite length. For bearings of finite length Gümbel introduced in the expression of the relation between X and $\eta\omega/k\psi^2$ the correction factor $c = 1 + d/l$ obtained by working up the test data of Lasche and his own data:

$$(X)* = \frac{1}{c}\frac{\eta\omega}{k\,^2}$$

The author found that the function (X) for a bearing of infinite length, as shown in figure 4 is satisfactorily expressed in terms of $\eta\omega/k\psi^2$ by the following empirical equation:

$$(X)* \doteq 1 + \frac{0.1}{\frac{\eta\omega}{k\psi^2}} \qquad (12)$$

For a bearing of finite length it is necessary in equation (12) to introduce the correction factor c. 829

Substituting equation (12) in expression (11) we obtain

$$f' = \frac{\pi}{\psi}\frac{\eta\omega}{k}\left(1 + \frac{0.1c}{\frac{\eta\omega}{k\psi^2}}\right) = \frac{\pi}{\psi}\frac{\eta\omega}{k} + 0.314c \tag{13}$$

that is, the first partial coefficient of friction is equal to the friction coefficient of Petroff for the concentrically rotating shaft plus a certain magnitude, constant for a given bearing, that expresses the effect of the eccentricity of the shaft in the bearing. It is very important to notice that the finite length of the bearing is expressed only through the constant term of equation (13). The smaller the value of l/d the greater for a given value of $\eta\omega/k$ the eccentricity of the shaft and the greater the coefficient of friction.

T" and d". We found previously (8a) that the friction force per unit area due to the flow in the loaded zone of the bearing is equal to:

$$\tau'' = \frac{h}{2}\frac{dp}{dx}$$

or in polar coordinates

$$\tau'' = \psi\,\frac{(1 + X\cos\varphi)}{2}\,\frac{dp}{d\varphi} \tag{14}$$

The equation of Reynolds, transformed for a cylindrical bearing, gives for the pressure gradient over the circumference of the bearing $dp/d\varphi$ the following expression:

$$\frac{dp}{d\varphi} = 6\eta\frac{\omega}{\psi^2}\,\frac{(1 + X\cos\varphi) - (1 + X\cos\varphi_0)}{(1 + X\cos\varphi)^3}$$

where φ_0 is the angle corresponding to the point of minimum clearance. It is simpler however to obtain the pressure gradient, with an accuracy sufficient for practical computations, if we make use of the test data on the limits of the bearing film and the character of the pressure change over the loading arc.

Numerous experimental investigations all give the following picture of the pressure distribution in the plane of symmetry of the full cylindrical bearing under constant load. The pressure, equal to zero over the entire periphery of the bearing (if the oil is fed without pressure), begins to increase over 90 to 120° from the line connecting the centers of the shaft and bearing, and increases with an almost constant gradient up to a maximum value lying near the point of narrowest clearance and constituting on the average 2.5 to 3 k. Thereafter the pressure drops sharply to zero or even somewhat below. A typical pressure curve measured by Bradford and Grunder (reference 10) is shown in figure 6.

Without great error this picture may be replaced by the scheme shown in figure 7. The pressure here increases linearly over an arc of 90°, attains the value 2.5 k at the point of minimum clearance, after which it drops very steeply. The assumed scheme is sufficiently near the true conditions, the approximation being closer the smaller the value of $\eta\omega/k$ and, as we shall see below, our results are of importance precisely for these values of $\eta\omega/k$. This assumption very much simplifies the computation of the coefficient of friction by equation (14). The pressure gradient $dp/d\varphi$ becomes constant and equal to $P_{max}/a = sk/a$ where a is the loading arc which in what follows we shall assume equal to $\pi/2$, $s = 2.5 - 3$.* For every other equivalent section of the bearing not lying in the plane of symmetry the magnitude $dp/d\varphi$ is equal to $2p_m/\pi$ where p_m is the pressure at the point of minimum clearance in that plane.

The pressure distribution over the length of the bearing (the Z axis) according to test data agreeing with the theoretical considerations (reference 11) is represented in any meridional section by a parabolic type curve with almost constant exponent m varying between 2.2 to 2.7. The maximum of the curve is on the axis of symmetry of the bearing as shown in figure 8 giving the pressure measurements of Nucker (reference 12) over the length of the bearing.

The pressure p_m may be expressed as a parabolic type function of the maximum pressure $p_{max} = sk$:

$$P_m = sk\left[1 - \left(\frac{z}{\frac{1}{2}}\right)^m\right]$$

*The computations made by the author for $a = 90$ to $120°$ show that the results given below are valid for the entire practically occurring range of values of a.

and the pressure gradient at any point of the loaded zone of the bearing may be expressed as follows: 829

$$\frac{dp}{d\varphi} = \frac{2p_m}{\pi} = \frac{2sk}{\pi}\left[1 - \left(\frac{z}{\frac{l}{2}}\right)^m\right]$$

The different points of the surface along the length of the bearing thus experience a different reaction τ'' as a function of their distance from the plane of symmetry of the bearing.

On the loaded surface of the shaft consider an element of width $rd\varphi$ and length dz (figure 9). The force dT'' acting on this element is equal to

$$dT'' = \tau'' \, rd\varphi \, dz = rd\varphi \, dz \, \psi \frac{(1 + X \cos \varphi)}{2} \frac{dp}{d\varphi}$$

$$= rd\varphi \, dz \, \psi \frac{(1 + X \cos \varphi)}{2} \frac{2sk}{\pi}\left[1 - \left(\frac{z}{\frac{l}{2}}\right)^m\right] \quad (15)$$

To determine the total force T'' acting on the shaft we integrate equation (15) twice, once over the length of the bearing, that is between the limits $\pm l/2$, and a second time over the length of the bearing film, that is between the limits π to $\pi/2$:

$$T'' = \frac{r\psi sk}{\pi} \int_{\frac{\pi}{2}}^{\pi} \int_{-\frac{l}{2}}^{+\frac{l}{2}} (1 + X \cos \varphi) \left[1 + \left(\frac{z}{\frac{l}{2}}\right)^m\right] d\varphi \, dz$$

$$= \frac{r\psi sk}{2} \frac{m}{m + 1} (1 - 0.64 X).$$

The friction coefficient f'' is equal to:

$$f'' = \frac{T''}{kld} = \frac{\psi s}{4} \frac{m}{m + 1} (1 - 0.64 X) \frac{\psi s}{4} \frac{m}{m + 1} (X)_\Delta$$

Assuming $s = 2.5$ and $m = 2.5$ we obtain

$$f'' = 0.4\psi(X)_\Delta$$

The function $(X)_\Delta$ expressed in terms of $\eta\omega/k\psi^2$ has the following form

$$(X)_\Delta = 1 - \frac{0.64}{1 + 2.5} \left(\frac{\eta\omega}{k\psi^2}\right)^{1.5}$$

The Relative Magnitudes of f' and f". The weight of each of the component parts of the total coefficient of friction is shown in tables 2 to 4. These tables give the values of f' and f" within the range $\eta\omega/k\psi^2 = 0.01$ to 100 which for the relative clearances $\psi = 0.001$, 0.002 and 0.0005 corresponds to the values of the operating parameter:

$$\lambda = 10 \div 100\ 000 \text{ centipoises} \times \text{rpm/kg/cm}^2$$

The double line in the tables divides off those values of $\eta\omega/k\psi^2$ which in all probability are outside the brakedown limits of the oil film. Examination of tables 2 to 4 shows that the term f" affects the total value of the coefficient of friction only in the range of small values of $\eta\omega/k\psi^2$ which probably lie outside the limits of fluid friction. For large values of $\eta\omega/k\psi^2$ the effect of the factor f" is not large. Computation shows that f" may without great error be assumed constant and equal to its mean value in the most important interval of $\eta\omega/k\psi^2$:

$$f'' = 0.25\psi$$

The error arising from this assumption in the least favorable case does not exceed 5 percent.*

Total Coefficient of Friction. Under the above made assumption the total coefficient of friction takes the following form:

$$f = f' + f'' \approx \frac{\pi}{\psi}\frac{\eta\omega}{k} + 0.314\psi + 0.25\psi = \frac{\pi}{\psi}\frac{\eta\omega}{k} + 0.55\psi \qquad (16)$$

The ratio l/d, as follows from the previous considerations, affects only the constant term of equation (16) as is fully confirmed by the above mentioned tests of McKee. According to

*The relative weight of f" in the general value of f has been computed on the assumption of the same viscosity of the oil over the loaded and nonloaded bearing. Actually the temperature of the oil in the loaded zone is always somewhat higher than in the remaining part due to the additional friction and the relative value of f" is somewhat smaller than that given in tables 2 to 4.

these tests for $l/d > 1$ the constant term of the expression for the friction coefficient must be corrected by the factor σ which may be expressed by the following equation:

$$\sigma = \left(\frac{d}{l}\right)^{1.5}$$

For values of $l/d > 1$ the correction factor σ is approximately equal to 1.

With the correction for the length of the bearing the friction coefficient takes the following form

$$f = \frac{\pi}{\psi} \frac{\eta\omega}{k} + 0.55 \left(\frac{d}{l}\right)^{1.5} \psi \tag{17}$$

or in practically suitable units:

$$f = 3.36 \cdot 10^{-9} \frac{d}{\Delta} \lambda + 0.55 \left(\frac{d}{l}\right)^{1.5} \frac{\Delta}{d} \tag{18}$$

where $\lambda = \eta n/k$

d the diameter of the shaft in mm

l the length of the bearing in mm

Δ the diametral clearance in mm

η the viscosity of the lubricant in centipoises

n the rpm

k the projected unit bearing load in kg/cm^2

The correction $(d/l)^{1.5}$ is introduced only for values of l/d less than unity.

Comparison with Test Data. Figure 10, to logarithmic scale, shows the curves of the coefficient of friction as systematised by A. Wewerka (references 16) according to the test results of a number of authors (references 13, 14, 15, 17) the formulas of Petroff, Gümbel and formula (17) of the present paper. The curves are plotted in the coordinates proposed by Howarth, f/ψ being laid off on the ordinate axis against $\eta\omega/k\psi^2$ on the abscissa.

The curves on figure 10 show clearly that the formula of Gümbel satisfactorily agrees with the test results for small values of $\eta\omega/k\psi^2$ but gives a large error for high values of this factor. The formula of Petroff, on the contrary, agrees very well with test results for large values of $\eta\omega/k\psi^2$ as was to be expected from the structure of the formula, but gives too small values for f for small values of $\eta\omega/k\psi^2$ for which the eccentricity of the shaft and the friction in the loaded zone have a large effort. Formula (17) satisfactorily agrees with test results over the entire range of values of $\eta\omega/k\psi^2$ practically encountered.

The dotted straight line drawn through the mean values of the coefficient of friction may serve for an orientating estimate of the coefficient of friction in the range $\eta\omega/k\psi^2 = 0.5$ to 100. The equation for this line is the following:

$$f = \frac{5}{\psi^{0.7}}\left(\frac{\eta\omega}{k}\right)^{0.85} \tag{19}$$

or in practical units

$$f = 12 \cdot 10^{-8}\left(\frac{d}{\Delta}\right)^{0.7} \lambda^{0.85} \tag{19}$$

where $\lambda = \eta n/k$,

η the viscosity in centipoises

n the rpm

k the projected unit bearing load in kg/cm^2

d the diameter of the shaft in mm

Δ the diametral clearance in mm

<u>General Character of Formula (17).</u> We may note that formula (17), derived on the assumption of constant load on the bearing, may be extended with a certain degree of reliability to other loading cases. Formula (17) without any reservations is applicable to the case of a shaft loaded by the rotating vector of a centrifugal load. This case represents the rotating system of a bearing under constant load with the characteristic feature that the load k is a function of the square of the speed and the coefficient of friction is a function of the speed and viscosity:

We may note that in this case the operating parameter drops with increased speed, in contrast to the case of constant load, and the coefficient of friction decreases with increased speed. For an impact load and for a load that varies in magnitude and direction formula (17) may serve as an approximation which is the more accurate the larger the value of $\eta\omega/k\psi^2$. The character and magnitude of the friction in the unloaded zone are here the same as in the case of constant load but the friction in the loaded zone has a different character. The curve of pressure in the loaded zone in all probability approximates a parabola the axis of symmetry of which for small speed of rotation passes through the line connecting the centers of the shaft and bearing and at large speed is displaced against the direction of motion. The oil in the loaded zone flows not only against the direction of motion as in the case of the constant load but also along the direction as a result of which a considerable decrease and even the total disappearance of the reaction of this flow on the shaft may be expected. The probable diagram of the velocities along the periphery of the shaft is shown in figure 11. On the other hand the relation between the load and the eccentricity of the bearing for an impact load and a load varying in magnitude and direction is other than for the case of a constant load and is determined not only by the parameter $\eta\omega/k$ but also by the rate of increase of the load and the rate of change of its direction. The eccentricity is in this case over a large part greater than in the case of a constant load.

Thus the variable and impact loads while leaving unchanged the first term of equation (17) change the constant term, the effect of the latter being in opposite directions. On the one hand the constant term decreases as a result of the flow in each direction in the loaded zone and on the other hand it increases as a result of the eccentricity. A detailed analysis of the friction for an impact and variable load constitutes a separate independent problem which the author hopes to consider another time.

Formula (17) may serve for an approximate estimate of the coefficient of friction in the case of an impact and variable load not only because of the opposite effect noted above the friction on the constant term of this equation but mainly because of the small relative value of this term in the practical interval of values of $\eta\omega/k$. As shown by tables 2 to 4 the constant term of equation (17) even for values of $\eta\omega/k$ near the critical does not exceed 25 percent of the total coefficient of friction. Actually bearings always work with a certain factor of safety and the magnitude of the constant term is generally very small in comparison with the first term.

APPLICATION OF THE OBTAINED RESULTS

1. kv as a Measure of the Thermal Stress of the Bearing. Formula (17) answers clearly the question often raised as to the applicability of the magnitude kv as a measure of the heat generated in the bearing. This method of estimating the thermal stress implicitly assumes the constancy of the coefficient of friction for all operating conditions and for various structural design factors of the bearings and at first view has no basis in fact if the friction theory based on Coulomb is rejected. Formula (17) shows that this is not at all the case and that under certain conditions the heat generation is actually proportional to kv. Nevertheless a careful analysis of the problem makes it necessary finally to reject the magnitude kv as well as any other magnitude even for a rough estimate of the thermal stress of the bearing.

The heat given out in unit time over unit surface of the bearing is expressed as a function of the coefficient of friction in the following manner:

$$\left(R\,\frac{\text{k cal}}{\text{sec}\cdot\text{m}^2}\right) = \frac{Pvf}{ld\;427} = \frac{kv}{427}\left(\frac{\pi}{k}\,\frac{\eta\omega}{k} + 0.55\,\sigma\psi\right)$$

$$= \frac{1}{427}\left(\frac{2\pi}{\Delta}\,\eta v^2 + 0.55\,\sigma\,\frac{\Delta}{d}\,kv\right) \qquad (20)$$

Thus the heat generated in the bearing consists of two parts: One part proportional to ηv^2 and the other to kv. The relative values of the two parts are shown in tables 5 and 6 where the magnitudes of the component parts of equation (20) are given as a function of $\eta\omega/k\psi^2$ for $l/d = 1$ and $\psi = 0.001$ and 0.002.

Tables 5 and 6 show that the magnitude kv predominantly determines the general value of the work of friction for small values of $\eta\omega/k\psi^2$ lying probably beyond the limits of fluid friction with the exception of those cases where the bearing has a small clearance and l/d is considerably less than 1. In the range of fluid friction the general value of R is greatly affected by the term proportional to ηv^2 and for large values of $\eta\omega/k\psi^2$ this term completely determines the value of R.

Thus the application of kv as a measure of the heat generation and as a basis for a comparative estimate of the thermal stress of the bearings may be justified for small values of $\eta\omega/k\psi^2$ approaching the critical value. A second condition is the equality of the relative clearance ψ and the ratio l/d for the bearings compared. For most bearings working with a sufficient safety factor the use of kv gives a large error. This however is not the main point. The determination of the load-carrying capacity of the bearing from the value of the heat generated involves a methodological error. The load-carrying capacity of the bearing is determined for a given load, speed and kind of oil by the temperature of the bearing which inturn is determined by the mutual interaction on the one hand of the heat generated and on the other the heat dissipated. The latter depends on a whole series of factors, for example, the size of the clearance, the feed pressure of the oil, the cooling surface, the number and arrangement of the oil grooves and the operating conditions characterized by the parameter $\eta\omega/k$.

On the basis of the above considerations the magnitude kv or any other magnitude must altogether be dispensed with for even a comparative estimate of the thermal stress of the bearing and the only rational way is to determine by computation the heat balance of the bearing and the effect of the heat generation and heat dessipation on the load-carrying capacity of the bearing. This will be done below.

2. Floating Bushes. Expression (17) sufficiently explains the advantages, often confirmed in practice under certain conditions, of floating bushes thus throwing doubt in the validity of the theories of the constancy of the coefficient of friction as shown below.

In a stationary bearing the heat generated is R = const Pvf. Now consider a floating bush. If the coefficient of friction is considered constant and it is assumed for simplicity that the floating bush rotates at a speed equal to half that of the shaft then on the two sides of the bush the amount of heat generated is

$$R' = 2 \text{ const } Pf\ v/2$$

that is, the floating bush gives no gain in the heat generated. We shall now apply equation (17) assuming that the bearing works in the region of high values of $\eta\omega/k$ so that the constant term in equation (17) may be neglected. The heat then generated in the stationary bearing is equal to

$$R' = \text{const} \quad \eta v^2$$

The heat generated on both sides in the floating bush (assuming the clearances the same) is

$$R' = 2 \text{ const } \eta \, v^2/4$$

that is, the floating bush lowers the heat generated by half. Two concentric floating bushes applied in certain especially high speed bearings lower the heat generation to one third. The smaller the value of $\eta\omega/k$ and the greater the weight of the constant term of equation (17) the less advantageous is a floating bush. If it is also remembered that the floating bush decreases the speed of rotation and hence the load bearing capacity by two the range of favorable application of floating bushes becomes entirely clear, namely, the range of high values of $\eta\omega/k$ of high speed, low load shafts working with a high factor of safety. In this case the floating bushes provide a technically rational means for lowering the temperature of the bearing though it is true at the expense of a lowering in the safety factor. We shall not here touch upon the other extreme of the favorable application of floating bushes, namely, in the range of half dry and half fluid friction.

3. Cut-away of Bearing and Shaft. A second method that has long been used in practice for decreasing the friction consists in increasing the clearance in the nonloaded part of the bearing as shown in figure 12a, or using a partial bearing as shown in figure 12b. For centrifugal load the same result is obtained by a cut-away of the shaft as shown in figure 12c. The advantages of the methods of figure 12a and 12c is twofold. In the first place the friction in the nonloaded region becomes negligibly small and in the second place the flow of oil is increased. This will be considered in detail in the section on the oil flow.

For partial bearings and cut away shafts and bearings the friction may be computed by the formula of Gümbel if the loading arc does not differ too greatly from 120°. Figure 10 permits the immediate determination of gain from the cut away of the shaft and bearing. For this purpose it is sufficient to compare the straight line representing the equation of Gümbel-Falz with the curve represented by formula (17) for a definite value of $\eta\omega/k\psi^2$. The gain is greater the greater the value of $\eta\omega/k\psi^2$. Thus according to figure 10 for $\eta\omega/k\psi^2 = 5$ the heat generated is reduced by half in comparison with the normal bearing and for $\eta\omega/k\psi^2 = 20$ it is decreased 3.5 times. For values $\eta\omega/k\psi^2 < 1$ the cut away of the shaft and bearing has no significance.

2. FLOW OF OIL FROM THE BEARING

The flow of oil from a full journal bearing with froced feed consists of two parts: (1) the flow q' from the loaded part under the action of the pressure developed in this zone, (2) the flow q'' under the action of the forced feed. 829

Determination of q'. For the solution of this problem we shall make use of the scheme, already applied by us, of the variation in pressure in the loaded part of the bearing, that is, we assume that the pressure in the loaded zone varies linearly over an angle of 90°.

The velocity of the oil along the Z axis (fig. 13) in the clearance of a bearing of height h according to equation (7) will be

$$v = \frac{1}{2\eta}\left[y^2 - \left(\frac{h}{2}\right)^2\right]\frac{dp}{dz}$$

The volume of oil flowing per unit time through the clearance of width b is equal to

$$q = b\int_{-h/2}^{+h/2} v\,dy = \frac{b}{2\eta}\int_{-h/2}^{+h/2}\left[y^2 - \left(\frac{h}{2}\right)^2\right]\frac{dp}{dz}\,dy = \frac{bh^3}{12\eta}\frac{dp}{dz} \tag{21}$$

We shall transform equation (21) into polar coordinates. We again turn to figure 3, taking on the surface of the shaft an element of width $r d\varphi$ and height $h = r$ (cos φ). The flow through this element in the direction of the Z axis is equal to

$$dq = \frac{r d\varphi\,(1 + X\cos\varphi)^3\, r^3\psi^3}{12\eta}\frac{dp}{dz} \tag{22}$$

As before we assume that the pressure along the axis of the bearing, that is, along the Z axis varies parabolically with exponent m. The pressure gradient dp/dz at the end of the bearing is equal to

$$\frac{dp}{dz} = \frac{mp}{\frac{l}{2}} \tag{23}$$

where p is the pressure in the plane of symmetry of the bearing. The variation of p over the range of the bearing angle on the basis of our mathematical assumptions may be expressed by the following equation:

$$p = p_{max}\left(\frac{\varphi - \pi}{a} + 1\right) = sk\left(\frac{\varphi - \pi}{a} + 1\right)$$

where $p_{max} = sk$ is the pressure at the point of minimum clearance in the plane of symmetry of the bearing, s is a coefficient fluctuating within the limits 2.5 to 3 and a the loading arc. For $a = \pi/2$ we have

$$p = sk\left(\frac{2\varphi}{\pi} - 1\right) \tag{24}$$

Substituting equation (24) in expression (23) and the latter in equation (22) we obtain the following expression for the flow through the element of the clearance in the loaded part of the bearing:

$$dq' = \frac{r^4\psi^3 msk}{6\eta l}\left(\frac{2\varphi}{\pi} - 1\right)\left(1 + X\cos\varphi\right)^3 d\varphi \tag{25}$$

The total flow over the entire loading arc is obtained by integrating equation (25) within the limits $\mu - \pi/2$:

$$q' = \frac{r^4\psi^3 msk}{6\eta l}\int_{\frac{\pi}{2}}^{\pi}\left(\frac{2\varphi}{\pi} - 1\right)(1 + X\cos\varphi)^3 d\varphi$$

$$= \frac{r^4\psi^3 msk}{6\eta l}\left(2.1 - 4.3X + 3.7X^2 - 1.1X^3\right) = \frac{r^4\psi^3 msk}{6\eta l}(X)_o$$

For the two sides of the bearing

$$q' = \frac{r^4\psi^3 msk}{3\eta l}(X)_o \tag{26}$$

The magnitude k is expressed as a function of the relative eccentricity X by equation (3):

$$k = \frac{\eta\omega}{\psi^2}\frac{1}{(X)_*}$$

where $(X)_*$ is the function represented in figure 1. Equation (26) in this case takes the following form:

$$q' = \frac{r^4\psi ms\omega}{3l}\frac{(X)_o}{(X)_*} \tag{27}$$

The magnitude $(X)_o/(X)_*$ is shown plotted in figure 14 as a function of X. As shown by figure 14 the flow from the loaded part of the bearing decreases with decreasing eccentricity of the shaft in the bearing and becomes equal to zero for a central position of the shaft. This was to be expected since for such position the bearing can not carry any load and there is no pressure in the oil film.

For $X = 1$ the flow is infinitely great since according to the Reynolds equation the pressure in the narrowest part of the clearance becomes infinitely great. Such a position never occurs in practice since long before this half-dry and dry friction arises as a result of the contact of the surfaces of the shaft and bearing.

The magnitude $(X)_o/(X)_*$ may with an accuracy sufficient for practical purposes be expressed as a function of $\eta\omega/k\psi^2$ by the following empirical equation:

$$\frac{(X)_o}{(X)_*} = \frac{0.25}{\left(\frac{1}{c}\frac{\eta\omega}{k\psi^2}\right)^{0.6}} \tag{28}$$

where c is the correction factor of Gümbel for length of bearing. Substituting expression (28) in equation (27) we obtain

$$q' = \frac{r^4\psi^{2.2}\ ms\omega\left(1 + \frac{d}{l}\right)^{0.6}}{12\ l\left(\frac{\eta\omega}{k}\right)^{0.6}}$$

Substituting

$$r = \frac{d}{2},\ \omega = \frac{\pi n}{30},\ s = 2.5\ m = 2.5$$

and replacing $(1 + d/l)^{0.6}$ by an approximate formula from the binomial expansion we obtain

$$q' = 0.0034\ \frac{d^3\psi^{2.2}n\sigma'}{\left(\frac{\eta\omega}{k}\right)^{0.6}} \tag{29}$$

where

$$\sigma' = \frac{d}{l}\left(1 + 0.6\ \frac{d}{l}\right)$$

The flow per rotation is equal to

$$qn' = 0.2 \frac{d^3\psi^{2.2}\sigma'}{\left(\frac{\eta\omega}{k}\right)^{0.6}} \tag{30}$$

Reducing equations (29) and (30) to practical units we obtain

$$q' \text{ cm}^3/\text{sec} = 0.8 \frac{d^{0.8}\Delta^{2.2}\sigma' n}{\lambda^{0.6}} \tag{31}$$

$$qn' \text{ cm}^3/\text{rotation} = 50 \frac{d^{0.8}\Delta^{2.2}\sigma'}{\lambda^{0.6}} \tag{32}$$

where $\lambda = \eta n/k$

d the diameter of the shaft in mm

Δ the diametral clearance in mm

η the viscosity of the oil in centipoises

n the rpm

k the projected unit bearing load in kg/cm^2

l the length of the bearing in mm

$\sigma' = \frac{d}{l}(1 + 0.6\ d/l)$

Equations (29) to (32) show that the flow of oil in the bearing without forced feed to a large extent depends on the clearance, on the ratio l/d, is almost directly proportional to the diameter of the shaft and inversely proportional to the parameter λ. For a given bearing the oil flow is a function of λ only and follows the law represented in figure 14.

The Bearing as a Stable System. Figure 14 illustrates the notable property of a slider bearing observable not only for a bearing without oil circulation but as we shall see later also for a bearing with forced circulation. This property is namely the fact that the bearing working in the range of fluid lubrication constitutes a system in equilibrium tending to maintain a definite value of the parameter $\eta\omega/k$ against all disturbing factors.

Let us compare figure 14 with the typical curve of the friction coefficient plotted against λ (fig. 15). We shall assume that the bearing works under a steady condition for example at $\lambda = 1500$. We shall assume further that some disturbing factor enters in the operation of the bearing, for example, an increase in the temperature due to local friction. In this case the viscosity of the oil drops and λ decreases. This produces on the one hand a decrease in the friction according to figure 15 and on the other hand an increase in the flow according to figure 14. The simultaneous action of these factors lowers the temperature and reestablishes the previous value of λ. An analogous picture is obtained for a decrease in λ due to and increase in k.

The reverse phenominon occurs on increasing λ. Let us assume for example that λ increases due to an increase in the speed. In this case the friction according to figure 15 increases, the flow simultaneously decreases and the temperature of the bearing rises as a result of which the viscosity of the oil decreases and the λ tends to drop.

The automatic character of this process is due to the property a of the oil of changing its viscosity with the temperature and the unique dependence of the flow on λ. In this peculiarity may be found the secret of the notable stability and noncapriciousness of behavior of correctly designed and constructed bearings. The essential condition for this automatic stability is that the bearing works with a sufficient safety factor and the time fluctuation of λ should not carry it beyond the critical values at which a breakdown of the oil film occurs.

Range of Application of Formulas (29) to (32). Equation (29) to (32) are applicable to the majority of technical cases of the bearings without forced feed lubrication or for a feed pressure not greater than 0.1 to 0.5 at. The essential condition for the applicability of formulas (29) to (30) is the absence of oil groves in the loaded zone and the rigidity of the bearing and shaft. Such grooves even though they do not extend to the ends of the bearing may increase the flow many times. However modern bearings only rarely have grooves in the loaded zone.

The deformation of the bearing or shaft under the action of the load may change the law of pressure distribution along the axis of the bearing and the quantity of oil flowing out.

Notwithstanding the fact that formulas (29) to (32) were derived on the assumption of constant load they may without reservations be applied to the case of a rotating centrifugal load which may be transformed into the scheme of constant load. It should be noted that the unit load k is here a function of the square of the speed and on increasing the speed the parameter λ, in contrast to the case of constant load, decreases and therefore the flow increases. This has long been observed on systems in which the centrifugal load predominated, for example, on crankshafts of internal combustion engines, etc.

In the case of an impact or variable load equations (29) to (32) may serve only as a first approximation since the law of pressure distribution along the loading arc in this case differs from that assumed as the basis of these equations which as a result give decreased values of q'.

<u>Oil Flow for a Bearing with Forced Feed Lubrication.</u> The oil flow for a bearing with forced feed depends on the number and arrangement of the oil feed openings and lends itself to analytical computation only in certain special cases one of which is chosen below.

We assume that the bearing in the plane of symmetry is provided with a ring groove in which oil is fed under pressure. We assume further, as above, that the pressure along the axis of the bearing is expressed approximately by a parabolic type curve the maximum ordinate of which is equal to the feed pressure p_0 (fig. 16). We shall compute the quantity of oil flowing from the clearance in the direction of the ends of the bearing, not considering for the present the oil flow from the loaded zone.

According to equation (21) the volume of oil q'' flowing per second from the two sides of the clearance is expressed as follows:

$$q'' = \frac{bh^3}{6\eta}\frac{dp}{dz}.$$

where h is the height of the clearance

b the width of the clearance

dp/dz the pressure gradient on the flow direction, that is along the Z axis.

We again turn to figure 3. For the element of width $r d\varphi$ and height

$$h = r\psi\,(1 + \chi\cos\varphi)$$

the flow per second is equal to

$$dq'' = \frac{r^4\psi^3\,(1 + \chi\cos\varphi)^3}{6\,\eta}\frac{dp}{dz} \tag{33}$$

829

By the equation of the parabolic curve

$$\frac{dp}{dz} = \frac{2mp_0}{l}$$

where m is the exponent of the curve, p_0 the feed pressure. Substituting this expression in equation (33) and integrating the latter between the limits 2π - 0, that is, over the entire periphery of the bearing, we obtain the total oil discharge per second:

$$q'' = \frac{r^4\psi^3mp_0}{3\eta l}\int_0^{2\pi}(1 + X\cos\varphi)^3\,d\varphi$$

$$= \frac{r^4\psi^3mp_0}{3\eta l}\,2\pi\,(1 + 1.5\,X^2) = \frac{2.1r^4\psi^3\,mp_0}{\eta l}\,(X)_\square$$

The function $(X)_*$ is plotted against $X_\square$ in figure 17. The latter shows that for full eccentricity of the shaft $(X = 1)$ the oil flow is two and a half times as large as for central position of the shaft $(X = 0)$. This was to be expected since according to equation (21) the flow very much depends on the amount of the clearance.

The oil discharge per rotation is equal to

$$q_n'' = 126\,\frac{r^4\psi^3mp_0}{\eta nl}\,(X)_\square$$

Expressing $n\eta$ in terms of (X) (equation (4)) we obtain

$$\eta n = \frac{30}{\pi}k\psi^2\,(X)*$$

where

$$q_n'' = 13.3\,\frac{r^4\psi^2mp_0}{kl}\,\frac{(X)_\square}{(X)_*} \qquad (34)$$

The function $(X)_\square/(X)*$ is shown plotted against X in figure 18 and with sufficient accuracy is expressed in terms of $\eta\omega/k\psi^2$ by the following empirical equation

$$\frac{(X)_\square}{(X)*}\,\frac{1.2}{\left(\frac{l\eta\omega}{ck\psi^2}\right)^{1.2}}$$

Substituting this expression in equation (34), setting $m = 2.5$ and expressing $(1 + d/l)^{1.2}$ by an approximate formula from the binomial expansion we obtain

$$q_n'' = 2.5 \frac{d^3\psi^{3.4}p_0\sigma''}{k\left(\frac{\eta\omega}{k}\right)^{1.2}} \tag{35}$$

where

$$\sigma'' = \frac{d}{l}\left(1 + 1.2\,\frac{d}{l}\right)$$

The flow is

$$q'' = 0.4\,\frac{d^3\psi^{3.4}p_0\sigma''}{\eta\left(\frac{\eta\omega}{k}\right)^{0.2}} \tag{36}$$

Reducing expressions (35) and (36) to practically suitable units we obtain

$$q'' = \text{cm}^3/\text{sec} = 2.5 \cdot 10^6\,\frac{\Delta^{3.4}\,p_0\sigma''}{d^{0.4}\eta\lambda^{0.2}} \tag{37}$$

$$q_n'' = \text{cm}^3/\text{rotation} = 1.6 \cdot 10^8\,\frac{\Delta^{3.4}\,p_0\,\sigma''}{kd^{0.4}\,\lambda^{1.2}} \tag{38}$$

where $\lambda = \eta n/k$

d the diameter of the shaft in mm

Δ the diametral clearance in mm

p_0 the feed pressure in kg/cm^2

η the viscosity in centipoises

n the rpm

k the projected unit bearing load in kg/cm^2

l - the length of the bearing in mm

$\sigma'' = \frac{d}{l}(1 + 1.2\, d/l)$

Formulas (35) to (38) show that the oil flow from the forced feed depends to a large degree on the clearance, the ratio l/d, is directly proportional to the feed pressure and inversely proportional to a small power of λ. Thus the flow of oil under the effect of forced feed, like the flow of oil from the loaded zone, is a factor of stability assuring the maintenance of a definite value of $\eta\omega/k$.

Range of Application of Formulas (35) to (38) Equations (35) to (38) are applicable to the case of forced oil feed from the end of the bearing except that in this case the values of q'' and q_n'' must be reduced to one fourth. In case the oil is fed through one or several holes on the periphery of the bearing equations (35) to (38) may be applied only as a first approximation. The more holes on the periphery and the closer they are arranged in the plane of symmetry of the bearing the more accurate the results that may be expected from formulas (35) to (38). In the case of a single hole or a small number of holes the actual discharge will be less than that computed by equations (35) to (38) by an amount which can be determined only experimentally.

If the holes are located in the lower loaded zone of the bearing (which rarely occurs in practice) there may be expected not only a decrease in the amount of oil flowing out but also a change of the dependence of the flow on λ. The shaft, as is known, changes its position in the bearing with change in λ, its center describing a path approximating a semicircle the top point of which coincides for infinitely large value of $\eta\omega/k$ with the center of the bearing. This displacement changes the cross-section of the oil holes and affects the flow of the oil.

The grooves and cut-aways in the bearing and shaft as shown in fig. 12a and 12c may increase the flow of the oil more than ten times the value computed by equations (35) to (38) and the discharge will not depend on λ. If the bearing has structural features having the object of increasing the oil circulation the flow of the oil through the clearance may be neglected and the oil outflow computed from the geometric dimensions of the grooves with the aid of the usual formulas of the flow of a viscous fluid.

Total Flow in Bearing with Forced Feed. The total flow of oil from a cylindrical bearing with forced feed is made up of the oil flowing out of the loaded zone and the oil flowing out under the effect of the feed pressure. As a first approximation we may neglect the effect of the oil feed grooves on the pressure distribution in the loaded region and consider the total flow of oil from the bearing as equal to the sum of Q' and q'' by equations (29) to (32) and (35) to (38). The total flow of oil in this case is expressed by the equation

$$Q \text{ cm}^3/\text{sec} = q' + q'' = 0.8 \frac{d^{0.8}\Delta^{2.2}\sigma' n}{\lambda^{0.6}} + \beta\, 2.5 \cdot 10^6 \frac{\Delta^{3.4} p_0\, \sigma''}{\lambda^{0.6}} \quad (39)$$

where β is a magnitude less than 1.

Comparison with Test Results. Formulas (29) to (32) and (35) to (38) were compared with the test results on oil flow published by Barnard (reference 17). The latter determined the flow of oil from a full cylindrical bearing of diameter 25.4 mm, 50.8 mm length with diametral clearance 0.15 to 0.28 mm. The feed pressure was varied between 0.7 to 4.9 kg/cm^2, the load between 2.9 to 19 kg/cm^2, the speed between 200 to 2000 rpm and the viscosity of the oil between 12.5 to 43.5 centipoises. The shaft was of hardened steel and the bearing of bronze. In the loaded part the bearing had a narrow oil distributing groove parallel to the axis of the bearing extending up to 5 mm from the ends. The oil flow through the bearing was expressed by the "useful pumping coefficient" E equal to the ratio of the actual flow of oil per rotation to the volume of the bearing clearance:

$$E = 2Qn/\pi d\Delta l$$

The results of the observations were grouped in the form of a set of curves of "useful pumping coefficient" as a function of the operating parameter $\eta n/k$ and the feed pressure p_0 (fig. 19). Figure 20 gives the values of the pumping coefficient computed by equations (32) and (38). Unfortunately Barnard does not show separately the values of n, η and k corresponding to the various values of λ whereas in the right member of our equation there enters the value of λ in addition to k. This difficulty was circumvented by assuming k inversely proportional to λ.

The "structural factor" of equations (32) and (38), equal respectively to $qn'/\lambda^{0.6}$ and $q_n''/\lambda^{1.2}$, was obtained by dividing the values measured by Barnard by λ. The agreement of figures 19 and 20 is more than satisfactory. It shows the correctness of the fundamental assumptions underlying formulas (29) and (32) and (35) and (38). Although our conclusions regarding the effect of λ on the oil flow through the bearing are entirely satisfied by the test results of Barnard the effect of the clearance on the oil flow according to our considerations is considerably greater than according to Barnard who did not at all observe the effect of the clearance on the useful pumping coefficient whence the conclusion may be drawn that the oil flow according to Barnard is directly proportional to the first power of the clearance. Formulas (32) and (38) however give a much sharper dependence. This point requires special investigation:

Comparison of q' and q". We shall consider the relative importance of the flow through the loaded zone q' and that due to the effect of the feed pressure q''. Dividing equation (28) by (32) and dividing out σ' and σ'' which do not greatly differ from each other we obtain

$$\frac{q_n''}{q_n'} = 31 \cdot 10^5 \psi^{1.2} \frac{p_0}{k} \frac{1}{\lambda^{0.6}}$$

Substituting the most frequently occurring value of the clearance $\psi = 0.001$ we obtain

$$\frac{q_n''}{q_n'} = 7700 \frac{p_0}{k} \frac{1}{\lambda^{0.6}}$$

We assume $k = 50 \text{ kg/cm}^2$ and shall vary the feed pressure p_0 between 1 to 6 at. The results of the computations for the various λ are given in table 7. Table 7 shows that for a smooth bearing and usual relative clearance $\psi = 0.001$ the flow under the effect of the forced feed considerable exceeds the "natural" flow of the bearing especially for small values of λ and large feed pressures. An increase in the clearance still further increases the part played by the forced flow.

Grooves, cut-aways, etc. in the unloaded zone may still more sharply change the relation between the "natural" and "forced" flows of the oil in favor of the latter and entirely mask the "natural" flow.

Range of Application of Obtained Results. We again mention the fact that formulas (29) to (32) and (35) to (38) are applicable only to the case of smooth shafts and bearings which do not deform under load for clearances of the order of $\psi = 0.002\text{-}0.0005$ for constant or centrifugal load. In the case of variable or impact load these formulas may be used only as a first approximation which is the more accurate the greater the part played by the "forced" flow in comparison with the "natural" flow. An analysis of the natural flow of oil for impact and variable loads requires the establishment of relations between the eccentricities of the shaft and the rate of change of the load, that is extension of the lubrication theory and its verification by experiment which, incidentally, is technically considerably more complicated than in the case of a constant load.

3. HEAT BALANCE OF THE BEARING

For a steady thermal state the amount of heat generated in the bearing is equal to that dissipated. The heat transfer is in the following three main parts: (1) in the oil flowing out of the bearing, (2) in the bush and body of the bearing, (3) in the shaft and parts connected with it. From the body and shaft the heat is removed to the surrounding atmosphere by convection and contact.

The two latter forms of heat transfer lend themselves with difficulty to mathematical computation. To a very large extent they depend on the special features of the bearing, the surfaces of the bearing and shaft, the number and shape of the structural details, etc. The heat transfer from the shaft, for example, increases sharply if there are rotating masses on the shaft, such as pulleys, couplings, propellers, etc. The heat transfer is greatly increased on the connecting rod journals of crankshafts in an air stream.

As far as is known to us the only attempt to compute this heat transfer (together with the heat given off by the oil) was made by Falz (reference 4). The computation is based on the tests of Lasche and is of a very primitive form. Lasche found that the heat transfer from the shaft of a bearing is proportional to the 1.3 power of the temperature difference between the bearing and cooling medium. The heat transfer equation according to Falz-Lasche has the following form:

$$R \text{ k cal/hr} = adl\,(t_n - t_a)^{1.3}$$

where

d is the diameter of the shaft

l the length of the bearing

t_n the temperature of the bearing

t_a the temperature of the surrounding medium

β_a a coefficient varying with the kind of bearing in the range 50 to 2000

For bearings of internal combustion engines Falz recommends choosing the values $a = 50$ to 2000 depending on the speed of the engine.

The exponent of $(t_n - t_a)$ which is somewhat greater than 1 shows that the most important part in the heat transfer of the bearing is played by convection and contact and not by radiation which is proportional to the difference between the fourth powers of the temperatures.

In practice a number of cases are known where the heat generated in the bearing is given off to the oil and the heat dissipation to the surrounding medium may be neglected. To such a case belong for example the bearings of an internal combustion engine of the airplane or automobile type enclosed in the crankcase and having a high temperature due to the contact with the heated working parts of the engine. The heat transfer from the bush of the bearing to the crankcase is in this case insignificant due to the small difference in temperature and the heat transfer from the shaft in the hollow of the crankcase is likewise small. An exception is possibly the case where the crankshaft is directly connected with a metal air propeller.

If we limit ourselves only to the heat dissipation in the oil then for a heat balance of the bearing under consideration the following equations hold.

The equation of the heat generated in the bearing:

$$R \text{ k cal/sec} = \frac{1}{427} Pvf = \frac{Pv}{427}\left(\frac{\pi}{\psi} \frac{\eta\omega}{k} + 0.55\,\sigma\psi\right) \tag{40}$$

The heat carried off by the oil in unit time:

$$R' \text{ k cal/sec} = c\, Q \frac{\gamma}{1000}(t_f - t_0) \tag{41}$$

where

c is the specific heat of the oil in k cal/kg° C

Q the flow of oil though the bearing in cm^3/sec

t_f and t_0 the final and initial temperature of the oil.

c and γ vary insignificantly with the temperature. The temperature of the bearing t_n may to a first approximation be considered equal to the mean temperature of the oil:

$$t_n = \frac{t_f + t_0}{2} = t_0 + \frac{\Delta t}{2},$$

where Δt is the increment of the temperature in the bearing. In this case equation (41) takes the following form:

$$R' = 2\ c\ Q \frac{8}{1000}(t_n - t_0)$$

The relation between the viscosity of the oil and the temperature is given by the curve of viscosity against t or by one of the many characteristic equations proposed by a number of authors. Thus according to Poiseuille (reference 9):

$$\eta = \frac{1}{A + Bt + Ct^2}$$

where A, B, C, are constants, characteristic for a given oil. According to Kiesskalt (reference 18):

$$\eta = \eta_0 b^{-\sqrt{t}}$$

where b is a constant. According to Falz (reference 4):

$$\eta = \frac{i}{(0.1t)^{2.6}}$$

where i varies from 1.06 for heavy oils to 0.07 for light oils. The author found that the viscosity of aviation oils (fig. 21) is well expressed by the following relation

$$\eta = \frac{i}{(0.1t)^3} \tag{42}$$

where i = 2.0 to 2.8 for heavy oils of the type of "brightstock" and 1.4 to 1.8 for castor oil and "kastrol".

For a steady thermal state the heat given out in the bearing is equal to the heat given to the oil:

629

$$R = R', \frac{Pv}{427}\left(\frac{\pi}{\psi}\frac{\eta\omega}{k} + 0.55\sigma\psi\right) = 2c\,Q\,\frac{8}{1000}(t_n - t_0) \qquad (43)$$

The simultaneous solution of equation (43) and the characteristic equation of the viscosity permits finding the temperature of the bearing for given speed, load and geometric dimensions.

We shall solve for example the following problem: It is required to find the temperature and safety factor of a bearing carrying 5000 kg at $n = 2000$ rpm. The dimensions of the bearing are $d = 100$ mm $l/d = 0.8$, $\Delta = 0.1$ mm, feed pressure of oil 3 at, temperature of oil 65°, the oil is "kastrol" with $i = 1.8$.

The unit load on the bearing is equal to:

$$k = \frac{5000}{80} = 63 \text{ kg/cm}^2$$

Assume that the temperature of the bearing is 90°. The viscosity of the oil is equal to 25 centipoises and λ is

$$\lambda = \frac{25 \cdot 2000}{60} = 800$$

The friction coefficient by equation (18) is equal to

$$f = 3.36 \cdot 10^{-9}\frac{1}{1000}\ 800 + 0.55\,\frac{\sqrt{100}}{80^{1.5}} \cdot 0.1 = 0.0034$$

The heat given off is

$$R = \frac{Pvf}{427} = \frac{5000 \cdot 10.4 \cdot 0.0034}{427} = 0.43 \text{ cal/sec}$$

The oil flow by equation (39) (for $\beta = 1$) is equal to

$$Q = 0.8\,\frac{d^{0.8}\Delta^{2.2}\,\sigma' n}{\lambda^{0.6}} + 2.5 \cdot 10^{6}\,\frac{\Delta^{3.4}p_0\sigma'}{d^{0.4}\eta\lambda^{0.2}} = 19.7 \text{ cm}^3/\text{sec}$$

Taking $c = 0.5$ and $\gamma = 0.85$ we obtain from equation (43):

$$0.43 = 19.7\,\frac{85}{100000}\,(t_n - 65)$$

$$t_n = 91^\circ$$

Thus no recomputation is required. The temperature of the oil passing out of the bearing is equal to

$$65 + 2\ (91 - 65) = 117^{\circ}\ C.$$

We shall find the factor of safety of the bearing for which the author (reference 19) proposes the ratio of the actual operating parameter λ to the critical parameter λ corresponding to the start of breakdown of the oil film. The critical value of the parameter is found from the limiting thickness of the oil film for which there is as yet no contact of the projecting roughnesses of the shaft and bearing. This magnitude depends mainly on the accuracy of the evaluation and the character of the surfaces of the shaft and bearing. We assume that the critical thickness of the oil film is 0.01 mm which corresponds to the eccentricity:

$$X = \frac{0.05 - 0.01}{0.05} = 0.8$$

From figure 5 for $\lambda/d = 0.8$ the corresponding value of the magnitude $\eta\omega/k\psi^2 = 0.36$ whence the critical λ is

$$\lambda = 9368 \cdot 10^5 \frac{0.36}{10^5} = 335$$

The safety factor of the bearing is equal to

$$x = \frac{\lambda}{\lambda\ kp} = \frac{800}{335} = 2.4$$

The safety factor can be increased in two ways: by an increase in the flow of oil through the bearing and by a decrease in the temperature of the oil feed. We shall first consider the first method. We shall raise the feed pressure to 6 at. The exact computation gives in this case $t_n = 87^{\circ}$. The viscosity of the oil is then equal to 27 centipoises, λ increases to 870 and the safety factor to

$$x = \frac{870}{335} = 2.6$$

We shall now attempt to lower the temperature of the oil feed to 50° at $p_0 = 6$ at. In this case the temperature of the bearing drops to 84° which corresponds to a viscosity of the oil of 31 centipoises, λ drops to 1140 and the safety factor to 829

$$x = \frac{1140}{335} = 3.4$$

There is noted the insignificant change in the temperature of the bearing for relatively large changes in the pressure and the temperature of the feed oil. This can be readily explained. By feeding a cooler oil we at the same time increase the friction as a result of which the heat generated in the bearing increases. On the other hand the flow of the oil decreases and the heat dissipation drops. A decrease in the inlet oil temperature even to as low a value as 20° gives a lowering in the temperature of the bearing only to 78°.

A much greater improvement is obtained by increasing the oil flow through an increase in the flow cross-sections in the unloaded zone, a method which at the same time reduces the friction. We assume that a cut-away is made in the bearing of height 0.5 mm on the unloaded bearing arc extending over 180°. The friction in this region may be approximately computed by the law of Newton:

$$T'' = \eta \frac{v}{h} \frac{\pi d l}{2} = \frac{\pi d^2 l}{2h} \eta\omega, \quad f'' = \frac{T''}{kld} = \frac{\pi d}{2h} \frac{\eta\omega}{k}$$

The friction in the loaded zone of the bearing may be approximately computed by the formula of Gümbel (reference 5):

$$f' = 1.7\sqrt{4\frac{d}{l} + 1}\sqrt{\frac{\eta\omega}{k}}$$

The total coefficient of friction is equal to

$$f = f' + f'' = 1.7\sqrt{4\frac{d}{l} + 1}\sqrt{\frac{\eta\omega}{k}} + \frac{\pi d}{2h} \frac{\eta\omega}{k}$$

The flow through the hollow (neglecting the effect of the eccentricity of the shaft) is found by formula (21):

$$Q = \frac{\pi d h^3 p}{6\eta l}$$

The flow through the loaded zone may be neglected. The temperature of the oil is determined from the equation of heat balance (43). Several successive approximations give:

$$t_n = 65 + 4 = 69^{\circ}$$

The viscosity is then equal to 60 centipoises, λ is equal to 1900 and the safety factor is

$$x = \frac{1900}{335} = 5.7$$

We shall lower the temperature of the inlet oil to 50°. The temperature of the bearing by the equation of heat balance is then obtained equal to 58.2°. To this corresponds a viscosity of 97 centipoises, λ is equal to 3100 and the safety factor is

$$x = \frac{3100}{335} = 9.2$$

This example shows that by a decrease in the temperature of the inlet oil a sharp increase in the safety factor and load bearing capacity of the bearing may be attained for a single constant condition namely an increased oil circulation obtained by structural means. If this condition is not satisfied the decrease in the temperature (and increase in the pressure) of the inlet oil does not give any essential result.

Load Bearing Capacity and Structural Factors. We shall consider the effect of structural factors on the temperature and load bearing capicity of the bearing with forced feed. For simplicity we shall assume that the bearing works at a high feed pressure so that the "natural" flow of the oil through the bearing may be neglected. We shall assume that the coefficient of friction varies according to equation (19). In this case the heat generated is equal to

$$R = \frac{Pvf}{427} = \frac{P\omega d}{2 \cdot 427} \frac{5}{\psi^{0.7}} \left(\frac{\eta\omega}{k}\right)^{0.85}$$

The oil flow is equal to

$$q'' = 0.4 \frac{d^3 \psi^{3.4} p_0 \sigma''}{\eta \lambda^{0.2}}$$

The heat transfer is

$$R' = q'' \frac{\gamma}{1000} (t_n - t_0)$$

The increment in the temperature of the bearing is determined from the equation 829

$$\Delta t = \text{const}\ \frac{(\eta\omega)^2 \left(\frac{l}{d}\right)}{\psi^4\ p_0} \tag{44}$$

The above equation shows that the temperature increment in the bearing is proportional to the square of the viscosity, the speed and the ratio l/d, inversely proportional to the fourth power of the relative clearance and ivnersely proportional to the feed pressure.

We shall now analyze the effect of the heat condition of the bearing on its load bearing capacity. We shall make use of the expression of Gümbel - Falz (reference 4, 19) based on the Reynolds equation for the load bearing capacity:

$$k = \text{const} \cdot \frac{\eta\,\omega \quad d^2}{h_{0\ min}\ \Delta c} \tag{45}$$

where $h_{0\ min}$ is the limiting allowable thickness of the oil film, $c = 1 + d/l$ the correction for the finite length of the bearing. Substituting the viscosity formula (40) in equation (45) we obtain:

$$k = \text{const}\ \frac{\omega\, d}{\left(t_0 + \frac{\Delta t}{2}\right)^3 \psi c} \tag{46}$$

Substituting in place of Δt its expression from (44) we obtain

$$K = \text{const}\ \frac{\omega\, d}{\left(t_0 + \frac{\Delta t}{2}\right)^3 \psi c} \tag{47}$$

The above equation shows that the predominating effect on the load bearing capacity is that of the bearing temperature. If no special means are provided for increasing the circulation of the oil the load bearing capacity is determined principally by the increase in the bearing temperature, the analysis of which was given above (formula (44)).

If forced circulation is used the temperature of the bearing is not large and the load bearing capacity is determined mainly by the temperature of the inlet oil so that all efforts of the designer should be aimed at a reduction of t_0. If the temperature of the bearing is given by the individual features of the bearing the load bearing capacity can only be affected by a change in the structural parameters and a choice of the kind of oil used.

COMPUTATION OF BEARING

The computation below refers to the special case where the heat given off to the surrounding atmosphere may be neglected and the main heat dissipation occurs in the oil.

The computation reduces to the determination of the temperature of the bearing with the aid of the equation of heat balance and the characteristic equation of the viscosity. From the temperature of the bearing is determined the mean viscosity of the oil after which by the known equations of the hydrodynamic theory it is determined whether the bearing can sustain the given load and with what safety factor. If too small values are obtained for the latter an increase in the load bearing capacity is obtained by one of the methods indicated above.

The clearance is rarely given as a definite figure but usually with a tolerance determined by the manufacturing possibilities. This circumstance and also the possible wear of the bearing and shaft must be taken into account in the computation in view of the considerable effect of the clearance on the friction and oil flow.

To increase the reliability of the computation the oil flow and coefficient of friction must be computed for the minimum tolerance and no wear. The oil feed, however, the cross-sections of the oil feed pipes and the output of the oil pump must be computed for the maximum tolerance and with wear taken into account. If the shaft and bearing are of different materials the amount of the clearance must be considered in the warm state taking account of the difference in the linear expansion coefficients of the materials.

Reference Data. In conclusion we present some data on the heat capacities and densities of oils at various temperatures.

Figure 22 shows the change of the densities of various aviation oils with temperature. The density may be determined by the formula

$$\gamma = \gamma_0 (1 - a\ t)$$

where γ_0 is the density at the initial temperature, a a constant lying between 0.0006x to 0.00070 and on the average equal to 0.00068.

The heat capacity of the oil is expressed by one of the following formulas:

$$c \frac{\text{k cal}}{\text{kg}^\circ\ \text{c}} = \frac{1}{\sqrt{\gamma}} (0.43 + 0.00081\ t), \quad c \frac{\text{k cal}}{} = \frac{A}{\sqrt{\gamma^{15}}} + B\ (t - 15)$$

where

γ_{15} is the density of the oil for 15° C

A a constant equal for neutral mineral oils to 0.405 to 0.425

B = 0.0009

REFERENCES

1. Petroff, N.: New Theory of Friction, Saint Petersburg 1883. Tests on the Friction of Fluids and Machines, Saint Petersburg 1886.

2. Gümbel I.: Die Reibungszahl der flüssigen Reibung under Berücksichtigung der endlichen Lagerbreite, "Zeit für das Ges. Turbinenwesens" 1918; Das Problem der Lagerreibung "Monatsblatter Berl. Bezirk VDI" May 1914, 1946.

3. Lasche O.: Die Reibungsverhältnisse in Lagern mit hohen Umfangsgeschwindigkeiten. "Zeit. VDI" Bd. 46, 1902, p. 1881.

4. Falz E.: Grundzüge der Schmiertechnik, 11. Auflage 1931, V-g. J. Springer, Berlin.

5. Illmer L.: Friction Coefficient Research, "Jr. SAE" Jan. 1930, vol. XXVI, N.1 p. 67.

6. Fomin, V.N. and Kokorev N. A.: Investigation of the Friction of a Light Engine, Moscow, 1931.

7. McKee, S. A. and McKee T. R.: Journal Bearing Friction as Influenced by Clearance and Length, "Trans. ASME" 1929, APM 51-15, p. 161.

8. Lamb H.: Hydrodynamics.

9. Gümbel L. and Everling: Reibung und Schmierung in Maschinenbau, Berlin, 1925.

10. Bradford I. J. and Grunder L. J.: Oil Film Pressures in a Complete Bearing "Tech. Bulletin" Eng. Exp. Station Pa. State College" N. 42, July 1933

11. Michell A. G. M.: The Lubrication of Plane Surfaces, Ostwalds Klassiker N. 218, Leipzig, 1927.

12. Nücker W.: Über den Schmiervorgang im Gleitlager, "Forschungsarbeiten VDI" N. 352, 1932, Berlin.

13. Köhler W.; Die Reibungsverhältnisse des Gleitlagers bei unvollkommener Schmierung, "Arch. fur d. Eisenhüttenwesen", Bd. 2, 1928 p. 159 - 168.

14. Schneider E.: Versuche uber die Reibung in Gleit - und Rollen Lager, Diss. Techn. Hochschule Karlsruhe, 1929, "Petroleum" Bd. 28, 1930, N. 7 and 11, pp. 221-236, 337-348.

15. Meyer-Jagenberg G.: Lagerversuche, "Werkst. - Technik" 1924, H. 3, 7, 8.

16. Wewerka A.: Berechnung und Konstruktion Zylindrischer Gleitlager nach neuen Unterlagen, "VDI", 28 Okt. 1933, N. 43. Bd. 77, pp. 1169-1170.

17. Barnard D. P.: The Mechanism of Lubrication, "Trans. SAE" v. 17, 1922, p. 203, Oil Flow in Plain Bearing, "Ind. and Engng. Chemistry", v. 18, May 1, 1926, N. 5, pp. 460-462.

18. Kiesskalt S.: Berechnung der Lagertemperatur, "Petroleum" Bd. 24, N. 27, Sept. 1928, pp. 1234-1235.

19. Orloff P. I.: On the Application of the Hydrodynamic Theory of Lubrication to the Computation of Aircraft Engine Bearings, "Technika Vosdushnogo Flota" No. 5, 1934, Moscow.

Translated by S. Reiss
National Advisory Committee
for Aeronautics.

TABLE I

Time of publication	Friction formula	Type of lubrication	d MM	$\frac{l}{d}$	ψ	Material	Speed range m/sec	Pressure range	Temperature °C	Viscosity range centipoises
1885, B. Tower	$f=0.0055 \frac{\sqrt{\eta v}}{k}$	Oil bath	101	1.5	1/1120	Steel on bronze	0.5-2.4	7-43	32	22-100
1902, Stribeck	$0.0016 \left(\frac{\eta}{k}\right)^{2/3} v^{1/3}$	Ring	70	1.95	1/950	Steel on babbitt	1.35-4	2-22	20-47	56-250
1902, O. Lasche	$0.0052 \frac{\sqrt{\eta} v^{1/3}}{k}$	Ring	260	0.42	1/950	Steel on babbitt	1-15	1-15	28-100	7-40
1913, C. Thomas, E. Maurer, L. Kelso	$0.002 \sqrt{\frac{\eta v}{k}}$	Ring	62	3.95	------	Steel on babbitt	0.5-1.4	2-7	27-43	40-100
1889, A. Martens	$0.00265 \frac{\sqrt{\eta v}}{k^{1/3}}$	--------	99	0.71	------	Steel on bronze	0.5-2	1-40	21-41	30-200
1903, A. Kingsbury	$0.00275 \left(\frac{\eta v}{k}\right)^{3/4}$	--------	35	1.45	------	Steel on bronze	0.2-0.5	2.8-2.4	32-77	15-46
1904, D. Woods and D. Carter	$0.003 \left(\frac{\eta v}{k}\right)^{1/3}$	--------	230	0.78	------	Steel on bronze	2.3-11	3.5-10.5	22-60	28-120
1922, D. Stoney, R. Boswall, D. Massey	$0.002 \frac{\eta^{0.44}\sqrt{v}}{k^{0.63}}$	Bath	63.5	1.6	------	Steel on babbitt	8.5-19	2.8-8.4	38-60	16-42
1922, G. Howarth and Nelson	$0.00096 \frac{\sqrt{\eta} v^{1/3}}{k^{2/3}}$	--------	50.8	0.23	------	Steel on bronze	3.4	28-56	40-90	10-30
1922, M. Hersey	$0.00115 \left(\frac{\eta v}{k}\right)^{3/4}$	--------	25.4	3	------	Steel on bronze	0.7-3	2.8-17.5	------	38-420
1923, G. Stanton	$0.0042 \frac{\sqrt{\eta} v^{1/3}}{k^{1/4}}$	--------	25.4	2.5	------	Steel on bronze	1.3	19	32-50	25-209
1928, D. Goodman	$0.0023 \frac{\eta^{0.37} v^{0.63}}{\sqrt{k}}$	Bath	152	0.5	------	Steel on bronze	0.75-3.5	2-15.4	16-72	40-54
1930, L. Illmer	$0.0033 \frac{\sqrt{\eta} v^{1/3}}{k^{3/4}}$	--------	-----	----	------	---------------	--------	0.2-7.5	20-150	10-400

TABLE II

$\psi = 0.001$

$\frac{\eta\omega}{k\psi^2}$	0.01	0.05	0.1	0.5	1	5	10	50	100
λ	10	50	100	500	1000	5000	10,000	50,000	100,000
$\frac{\pi}{\psi} \cdot \frac{\eta\omega}{k}$	0.00003	0.000157	0.000314	0.00157	0.00314	0.0157	0.0314	0.157	0.314
In o/o of f	6.3	25	39.3	73.4	83	95.7	97.8	99.5	99.8
0.314 ψ	0.000314	0.000314	0.000314	0.000314	0.000314	0.000314	0.000314	0.000314	0.000314
In o/o of f	63.2	50	39.3	14.6	8.3	9.1	1	0.2	0.1
f"	0.00015	0.00016	0.00017	0.00026	0.00034	0.00039	0.00039	0.00041	0.0004
In o/o of f	30.5	25	21.3	12	8.7	2.4	1.2	0.3	0.1
	0.00049	0.00063	0.0008	0.0021	0.0038	0.0164	0.032	0.1577	0.3147

TABLE III

$\psi = 0.002$

$\frac{\eta\omega}{k\psi^2}$	0.01	0.05	0.1	0.5	1	5	10	50	100
λ	40	200	400	2000	4000	20,000	40,000	200,000	400,000
$\frac{\pi}{\psi} \cdot \frac{\eta\omega}{k}$	0.0000628	0.000314	0.000628	0.00314	0.00628	0.0314	0.0628	0.314	0.628
In o/o of f	6.3	25	39.3	73.4	83	95.7	97.8	99.5	99.8
0.314 ψ	0.000628	0.000628	0.000628	0.000628	0.000628	0.000628	0.000628	0.000628	0.000628
In o/o of f	63.2	50	39.3	14.6	8.3	1.9	1	0.2	0.1
f"	0.0003	0.00032	0.00034	0.00052	0.00068	0.00078	0.00078	0.0008	0.0008
In o/o of f	30.2	25	21.3	12	87	2.4	1.2	0.3	0.1
f	0.00099	0.00126	0.00150	0.00429	0.00759	0.0328	0.0642	0.315	0.629

TABLE IV

$\psi = 0.0005$

$\frac{\eta\omega}{k\psi^2}$	0.01	0.05	0.1	0.5	1	5	10	50	100
λ	----	----	25	125	250	1250	2500	12,500	25,000
$\frac{\pi}{\psi} \cdot \frac{\eta\omega}{k}$	----	----	0.000157	0.000785	0.00157	0.00785	0.0157	0.0785	0.157
In o/o of f	----	----	39.3	73.4	83	95.7	97.8	99.5	99.8
$0.314\ \psi'$	----	----	0.000157	0.000157	0.000157	0.000157	0.000157	0.000157	0.000157
In o/o of f	----	----	39.3	14.6	8.3	1.9	1	0.2	0.1
f"	----	----	0.000085	0.00013	0.00017	0.00019	0.0002	0.0002	0.0002
In o/o of f	----	----	21.4	12	8.7	2.4	1.2	0.3	0.1
f	----	----	0.0004	0.00107	0.0019	0.0082	0.016	0.788	0.1573

TABLE V

$\psi = 0.001$

$\frac{\eta\omega}{ck\psi^2}$	0.01	0.1	1	10	1000
Const ηv^2	0.0000314	0.000314	0.00314	0.0314	0.314
o/o of R	5.4	36	85	98.39	99.83
Const kv	0.00055	0.00055	0.00055	0.00055	0.00055
In o/o of R	94.6	64	15	1.7	0.17
R	0.0005814	0.00086	0.00369	0.03195	0.3145

TABLE VI

$\psi = 0.002$

$\frac{\eta\omega}{k\psi^2}$	0.01	0.1	1	10	100
Const ηv^2	0.0000157	0.000157	0.00157	0.0157	0.157
In o/o of R	1.14	12.5	59	93.5	99.3
Const kv	0.0011	0.0011	0.0011	0.0011	0.0011
In o/o of R	98.6	87.5	41	6.8	0.7
R	0.001115	0.001257	0.00267	0.0168	0.1581

TABLE VII

λ		250	500	1000	5000	10,000
$\frac{q_n''}{q_n'}$	p_o = 1 at.	5.5	3.8	2.55	0.95	0.6
	3 at.	17	11.5	7.7	2.9	1.8
	5 at.	27.5	19	12.4	4.8	3

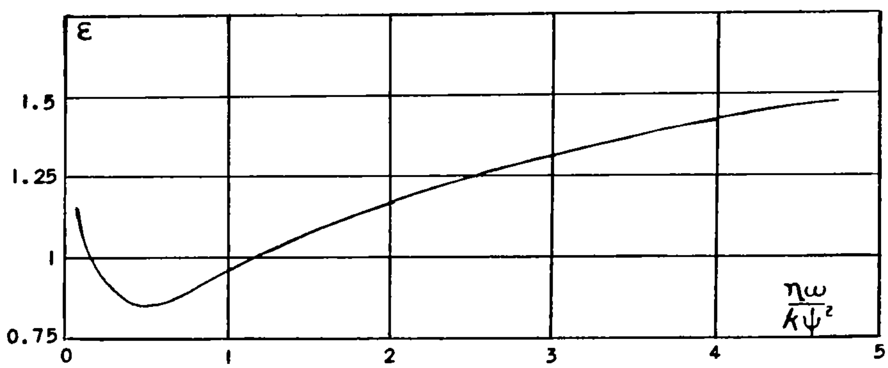


Fig. 1

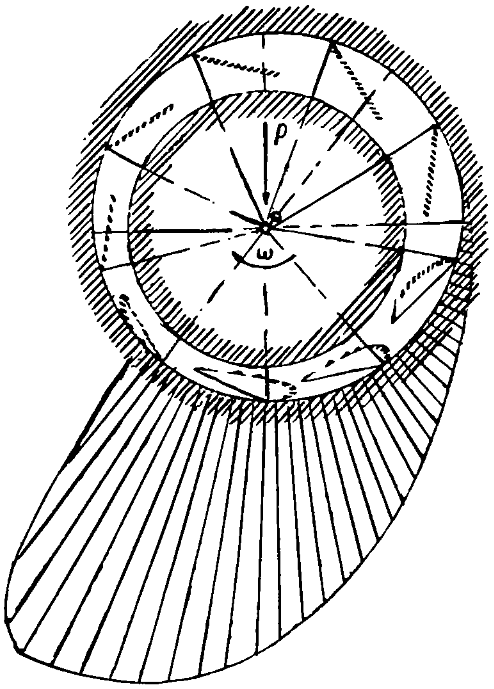


Fig. 2

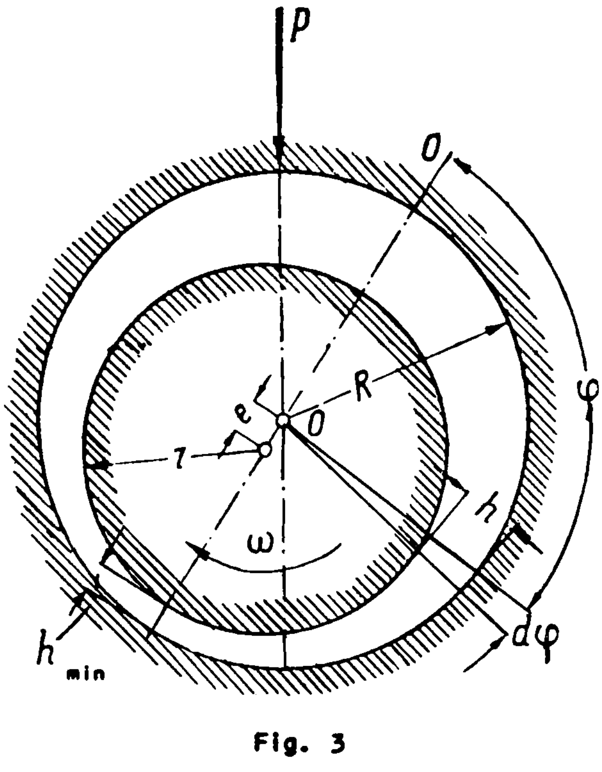


Fig. 3

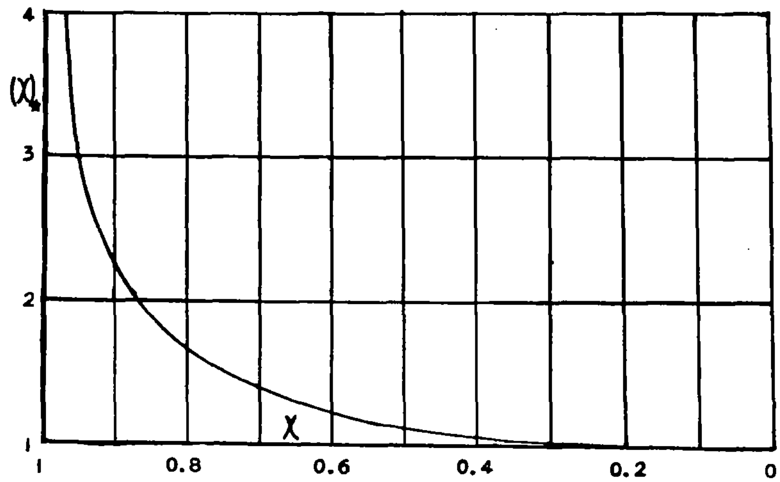


Fig. 4

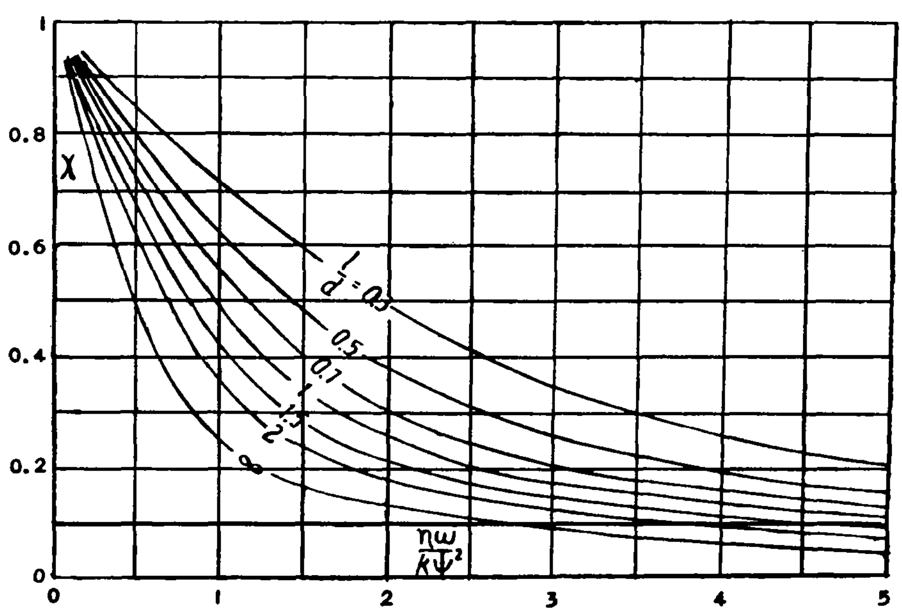


Fig. 5

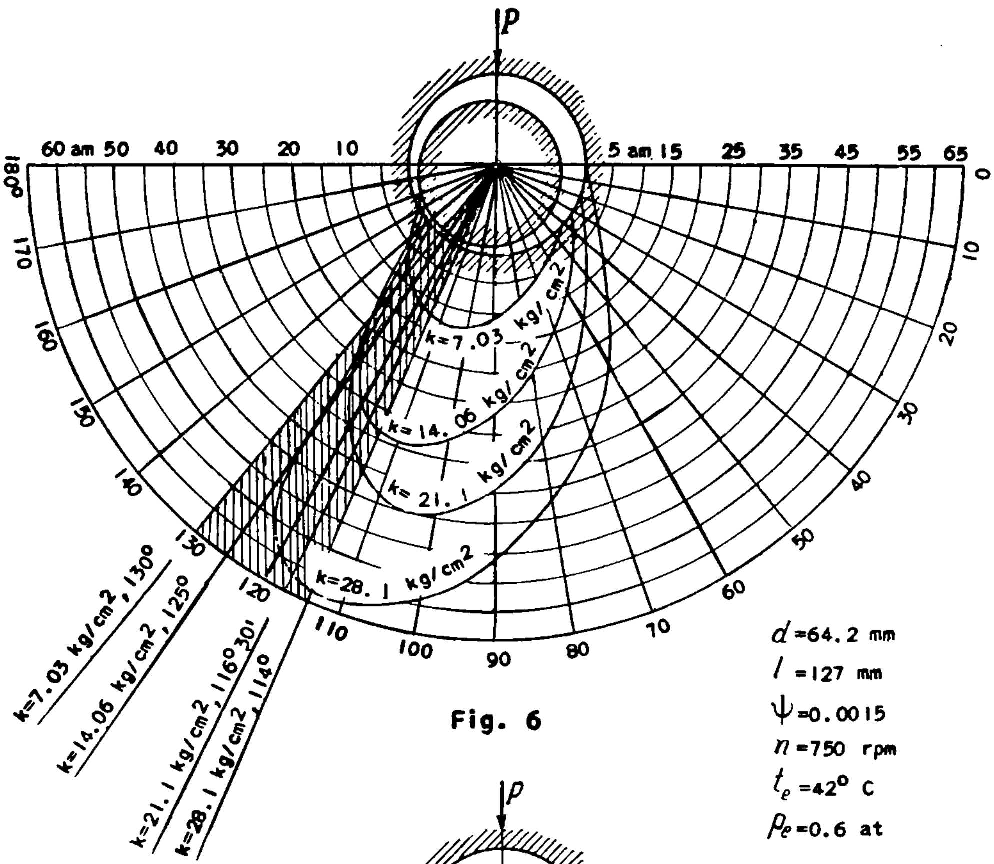

P
k=7.03 kg/cm2
k=14.06 kg/cm2
k=21.1 kg/cm2
k=28.1 kg/cm2
k=7.03 kg/cm2, 130°
k=14.06 kg/cm2, 125°
k=21.1 kg/cm2, 116°30'
k=28.1 kg/cm2, 114°
d=64.2 mm
l=127 mm
ψ=0.0015
n=750 rpm
te=42° C
pe=0.6 at

Fig. 6

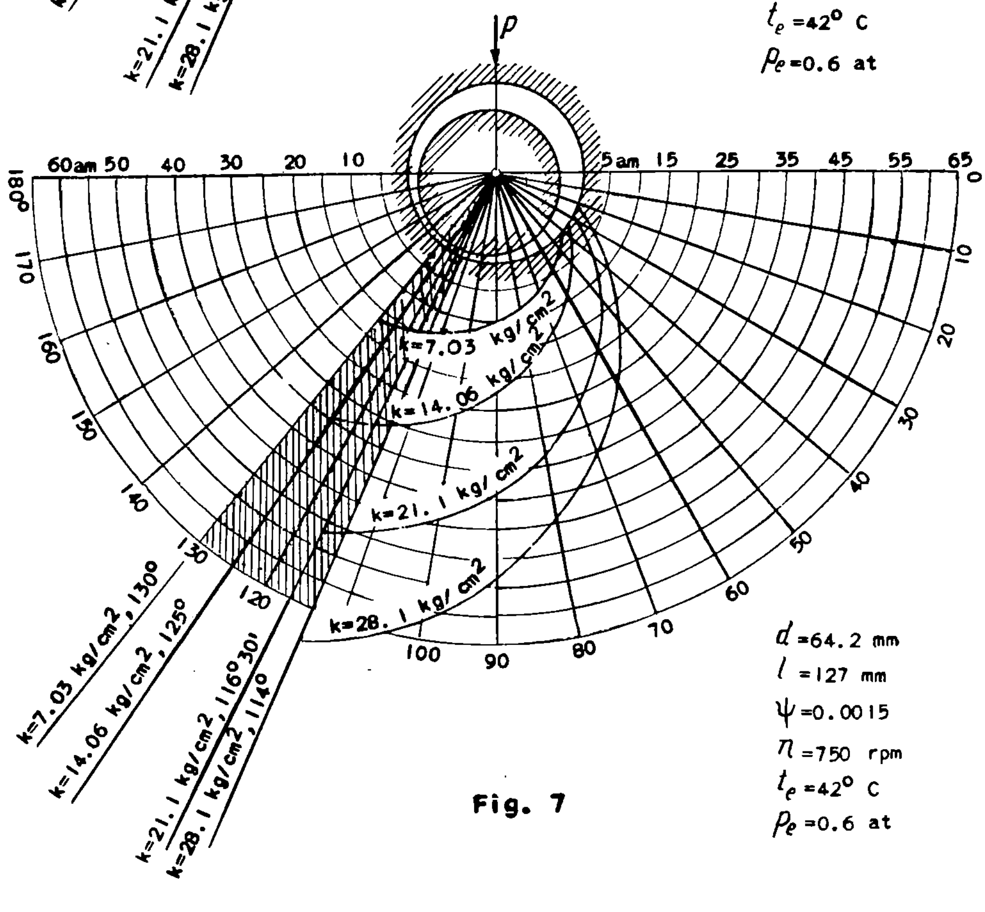

P
k=7.03 kg/cm2
k=14.06 kg/cm2
k=21.1 kg/cm2
k=28.1 kg/cm2
k=7.03 kg/cm2, 130°
k=14.06 kg/cm2, 125°
k=21.1 kg/cm2, 116°30'
k=28.1 kg/cm2, 114°
d=64.2 mm
l=127 mm
ψ=0.0015
n=750 rpm
te=42° C
pe=0.6 at

Fig. 7

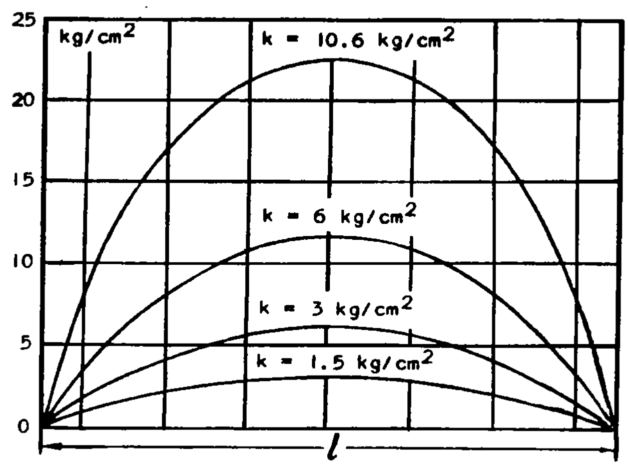

25
20
15
10
5
0
kg/cm2
k = 10.6 kg/cm2
k = 6 kg/cm2
k = 3 kg/cm2
k = 1.5 kg/cm2
l

Fig. 8

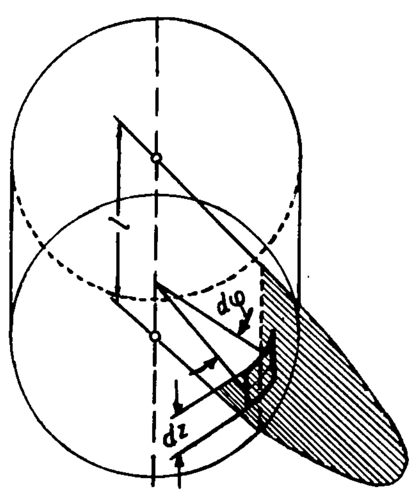
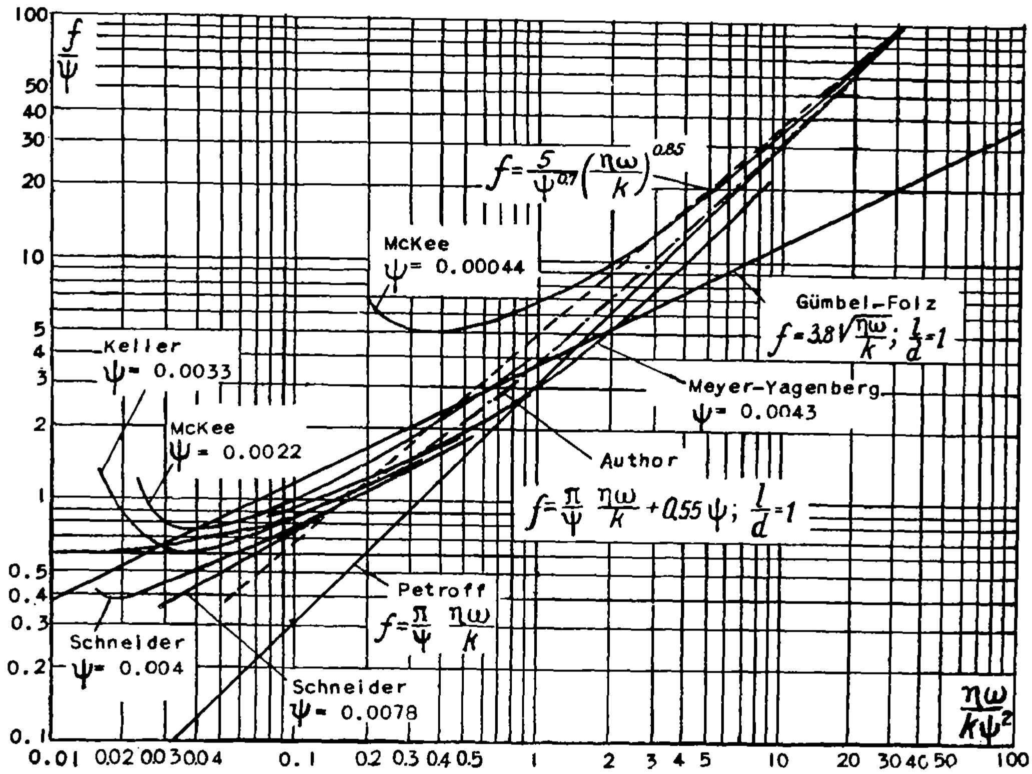

l
dφ
dz

Fig. 9



$\frac{f}{\psi}$
$f=\frac{5}{\psi^{0.7}}\left(\frac{\eta\omega}{k}\right)^{0.85}$
McKee
ψ= 0.00044
Gümbel-Folz
$f=3.8\sqrt{\frac{\eta\omega}{k}};\ \frac{l}{d}=1$
Keller
ψ= 0.0033
Meyer-Yagenberg
ψ= 0.0043
McKee
ψ = 0.0022
Author
$f=\frac{\pi}{\psi}\ \frac{\eta\omega}{k}+0.55\psi;\ \frac{l}{d}=1$
Petroff
$f=\frac{\pi}{\psi}\ \frac{\eta\omega}{k}$
Schneider
ψ= 0.004
Schneider
ψ= 0.0078
$\frac{\eta\omega}{k\psi^2}$

Fig. 10

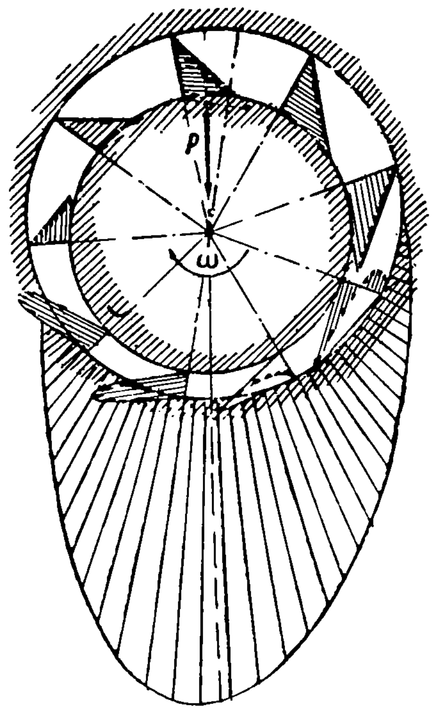


Fig. 11

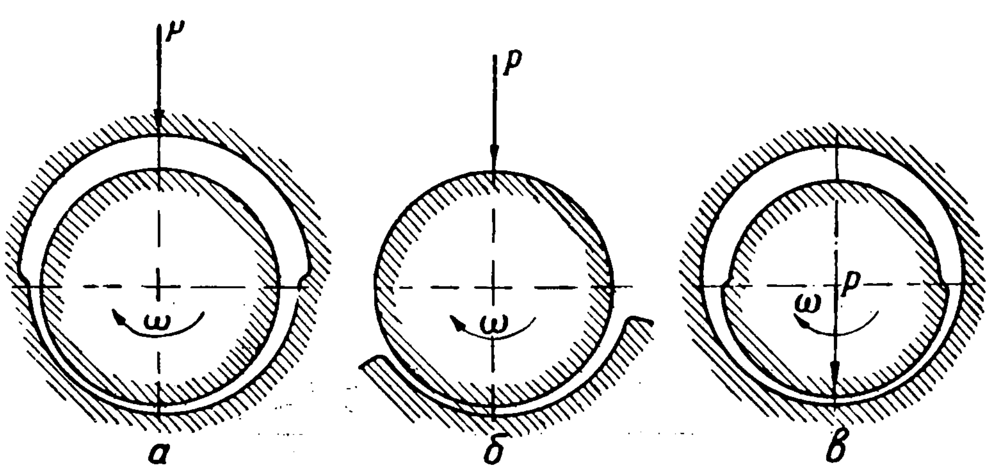


Fig. 12

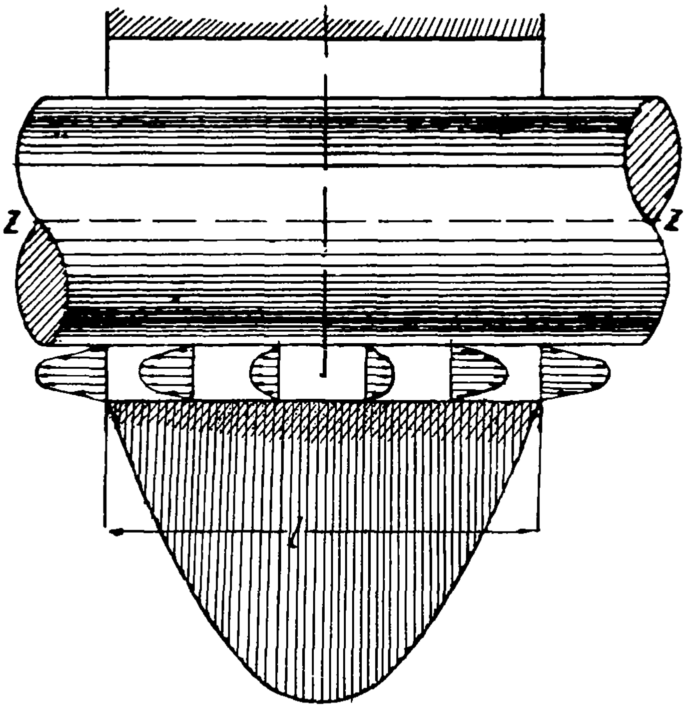


Fig. 13

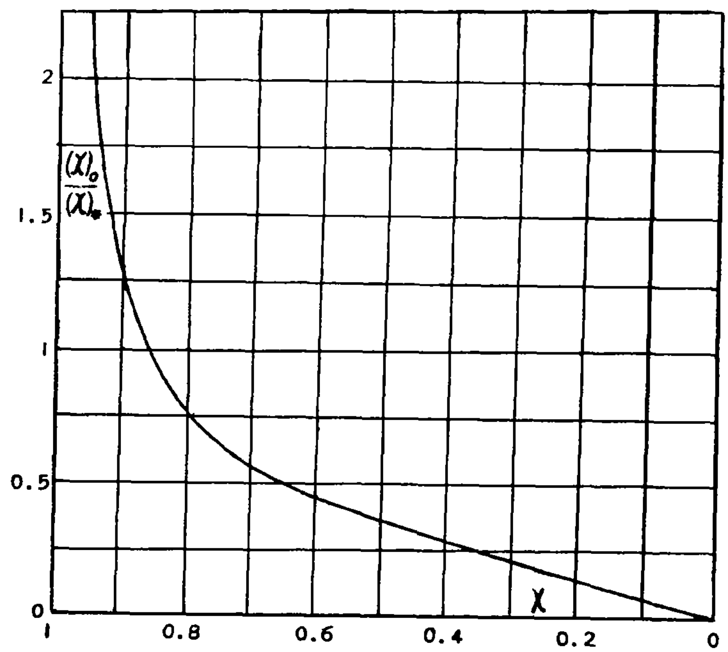


Fig. 14

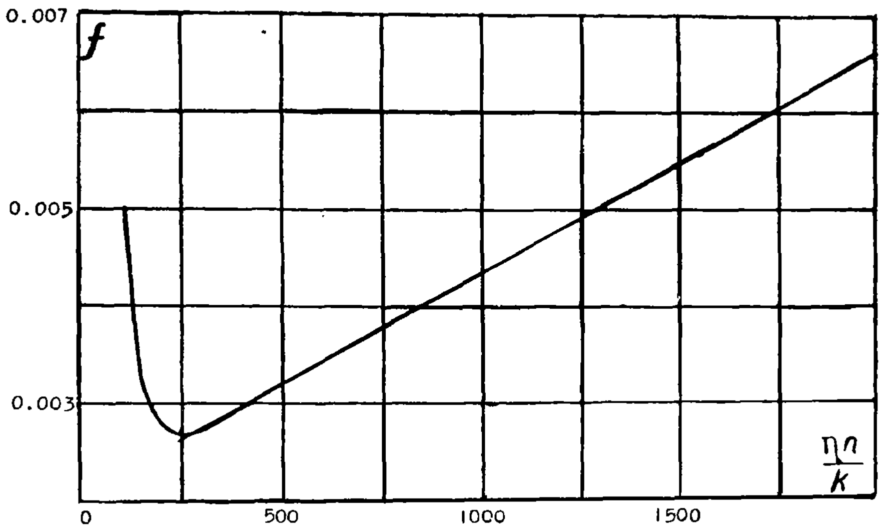


Fig. 15

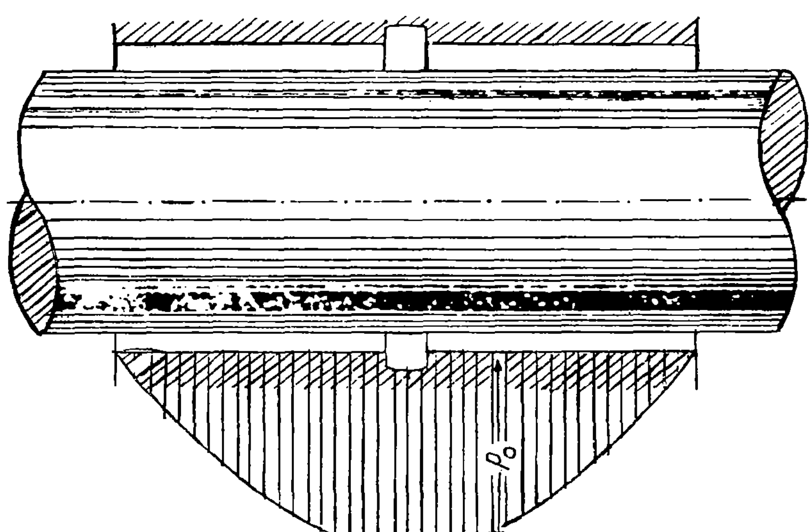


Fig. 16

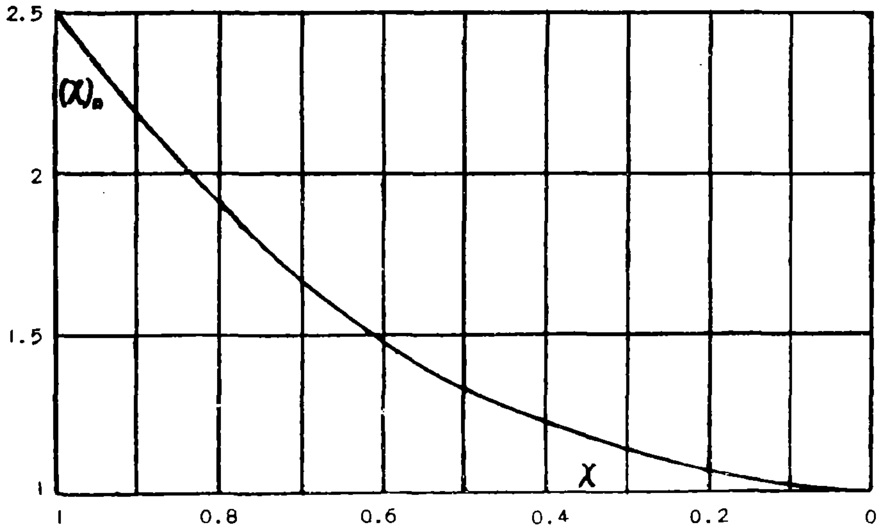

2.5
$(\chi)_a$
2
1.5
1
χ
1
0.8
0.6
0.4
0.2
0

Fig. 17

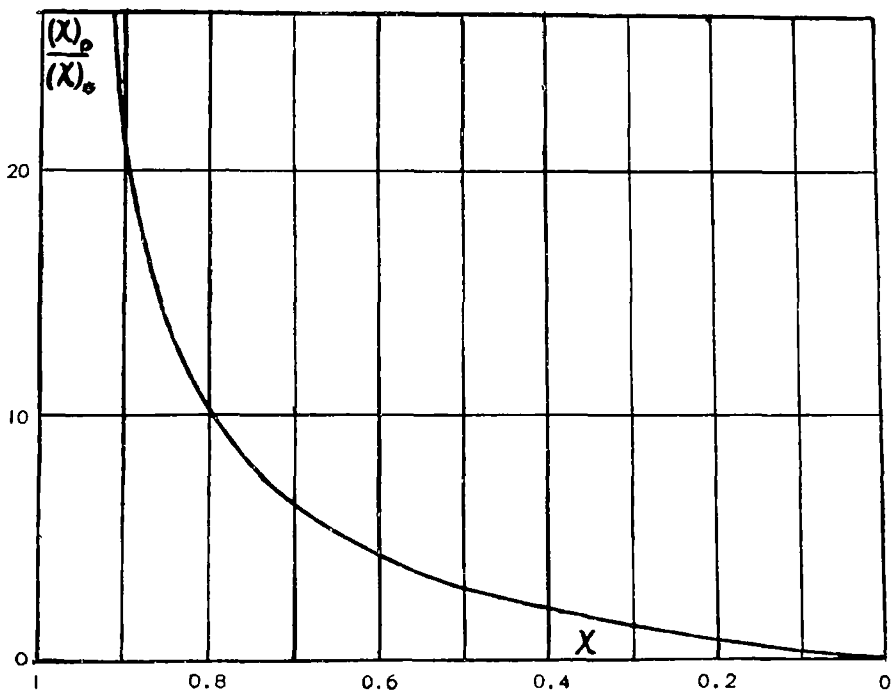

$\frac{(\chi)_p}{(\chi)_a}$
20
10
0
χ
1
0.8
0.6
0.4
0.2
0

Fig. 18

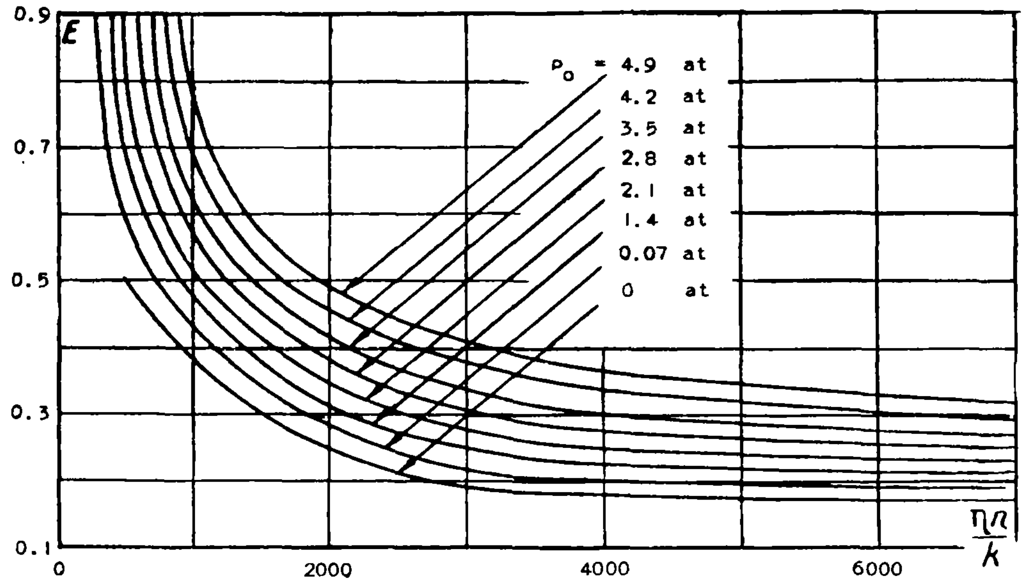

E
P_o = 4.9 at
4.2 at
3.5 at
2.8 at
2.1 at
1.4 at
0.07 at
0 at
0.9
0.7
0.5
0.3
0.1
0
2000
4000
6000
$\frac{\eta n}{k}$

Fig. 19

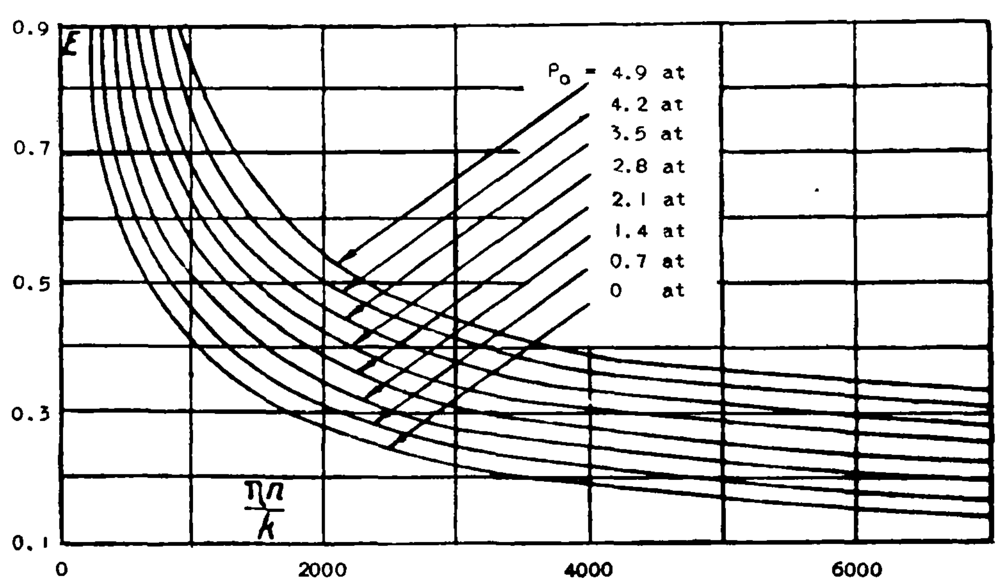

E
P_o = 4.9 at
4.2 at
3.5 at
2.8 at
2.1 at
1.4 at
0.7 at
0 at
0.9
0.7
0.5
0.3
0.1
0
2000
4000
6000
$\frac{\eta n}{k}$

Fig. 20

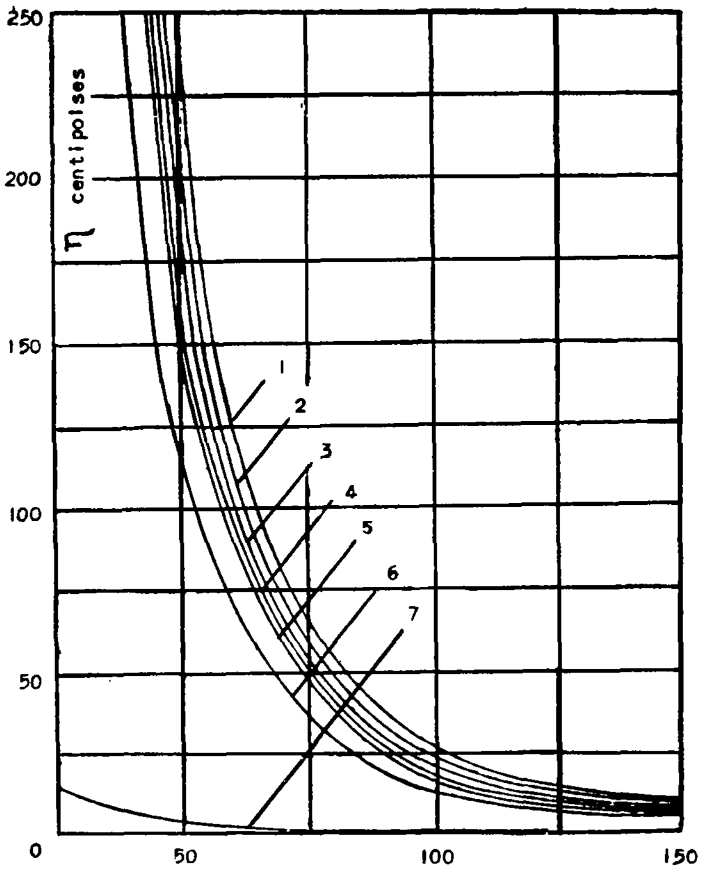


Figure 21. - Notation: 1-"Embinsky Brightstock", 2-aviation 19, 3-AAC aviation, 4-"Surochansky Brightstock", 5-"Castrol", 6-castor oil, 7-spindle oil.

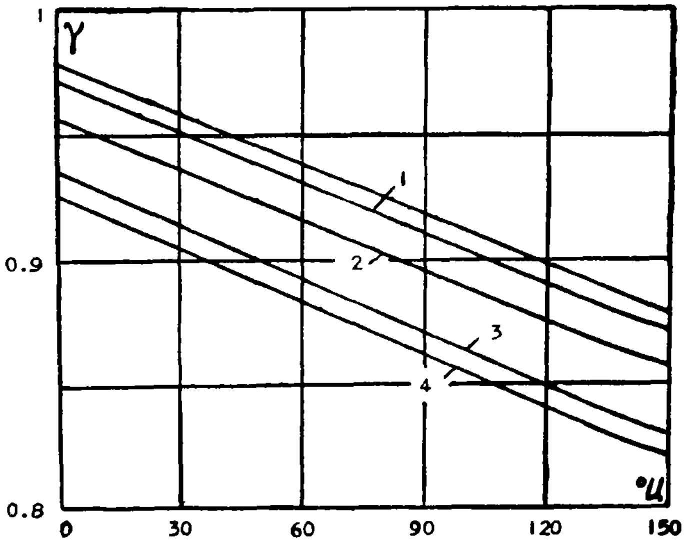


Figure 22. - Notation: 1-castor oil, 2-aviation 19, 3-"Embinsky Brightstock", 4-"Castrol".